COMPUTER SCIENCE

METHODS MODELING SYSTEMS FOR THE IMPROVEMENT OF THEIR RELIABILITY

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ABSTRACT

The method of Markov's processes for the analysis of systems with constant bounce and recovery intensities considered. The article presents calculations of the failure probability of the system for describing the various cases of redundancy of its components using Markov's models. Expressions obtained for calculating the approximate value of the failure probability of the system and analyzed of failures to improve the reliability of the system. The Markov's graph of transitions in the reservation of the system, which reflects its behavior, described. Analysis of the results of numerical solution of systems shows that when loaded with redundancy, the probability of failure is higher than with partially loaded, and with partially loaded - higher than with unloaded backup. A tree of errors for the system of cooling and clearing of flue gas at the reservation made by replacing "2 of 3", which has seven minimum bounce cross sections. Calculated the probability of system failure. The obtained calculations allow to analyze failures of technical systems in order to increase the reliability of their functioning.

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Introduction. A scientific approach to safety concerns requires a comprehensive analysis and classification of man-made accidents, major environmental and environmental factors, environmental behavior and personnel actions. Appropriate mathematical modeling methods and physical models of accident occurrence and development are required to address these issues.

The reliability and security indicators of the system include quantitative reliability characteristics, which are introduced and determined according to the rules of statistical theory of reliability, probability theory and mathematical statistics.

The choice of the method of reliability analysis of renewable redundant systems is largely determined by the class of the constructed reliability model. Currently, the main classes of models under study in reliability theory are independent event models, Markov and semi-Markov models. For their analysis, methods of probability calculation, methods of theory of Markov random processes, as well as methods based on the addition of differential equations are used.

Due to the comparative simplicity and clarity of the mathematical apparatus, the high probability and accuracy of the obtained decisions, Markov processes are of particular interest in risk assessment and design of decision support systems.

Standard approaches for reliability are based on a probabilistic model, which is often inappropriate for tasks of this kind [1, 2]. Probability theory is often a complex and not intuitive approach, the result of which is difficult to analyze. Similarly, probabilistic analysis usually requires more information about the system than it is known about, for example, the distribution of failure rates [3]. Typically, this leads to false assumptions about the raw data. The probabilistic paradigm also has many limitations when applied to small-volume samples [4].

Analysis of the development of technical systems allows us to conclude that, despite the rapid development of such areas as systems theory, including theory of automatic control, theory of reliability, theory of security, to describe the behavior of complex systems of existing mathematical models and methods is not enough. This position is clearly reflected in the research of A.V. Akimova, M.A. Yastrebenets′koho, H.M. Druzhynina [5-7]. Also, the risk of technical systems at different times was addressed by such researchers as M. Rasmussen, O.Renn, B.V. Hnedenko, I.A. Ryabinin et al. They noted that it was impossible to ignore this area of study of the security of technical systems.

Approaches using failure trees and their varieties are well adapted to analyze the reliability of technical systems, but they are somewhat limited in application to real complex systems.

Partial failures, coverage, system serviceability, and other important reliability issues are well covered by the failure tree analysis method [8]. An alternative to this approach is to use Markov processes. However, a review of their models showed that they were not sufficiently investigated in the problems of reliability of technical systems.

Purpose of the study is to perform system failure calculations to describe the various cases of redundancy of its components using Markov models. Obtain expressions to calculate the approximate value of the system failure probability and analyze the failures to improve the reliability of the system.

Research results. Consider the Markov process method for analyzing systems with constant failure rates and recoveries ($\lambda -$ conditional failure flow rate, $\mu -$ conditional recovery flow rate).

Let $x(t) = 1,$ if the component is inoperable and $x(t) = 0,$ if the component is inoperable.

The following expression system can be used to determine the conditional failure rate:

$$\begin{aligned} P(1|0) &\equiv \Pr\left[x(t+\Delta t) = 1 | x(t) = 0\right] = \lambda \Delta t; \\ P(0|0) &\equiv \Pr\left[x(t+\Delta t) = 0 | x(t) = 0\right] = 1 - \lambda \Delta t; \\ P(1|1) &\equiv \Pr\left[x(t+\Delta t) = 1 | x(t) = 1\right] = 1 - \mu \Delta t; \\ P(0|1) &\equiv \Pr\left[x(t+\Delta t) = 0 | x(t) = 1\right] = \mu \Delta t, \end{aligned} \tag{1}$$

where $\Pr\left[x(t+\Delta t) = 1 | x(t) = 0\right]$ is the probability that the failure will occur within the time interval $t + \Delta t$, provided that the component is operable at time t, etc.

Values $P(1|0), P(0|0), P(0|1)$ are called transition probabilities [9, 10] (transitions between states are shown in Fig. 1).

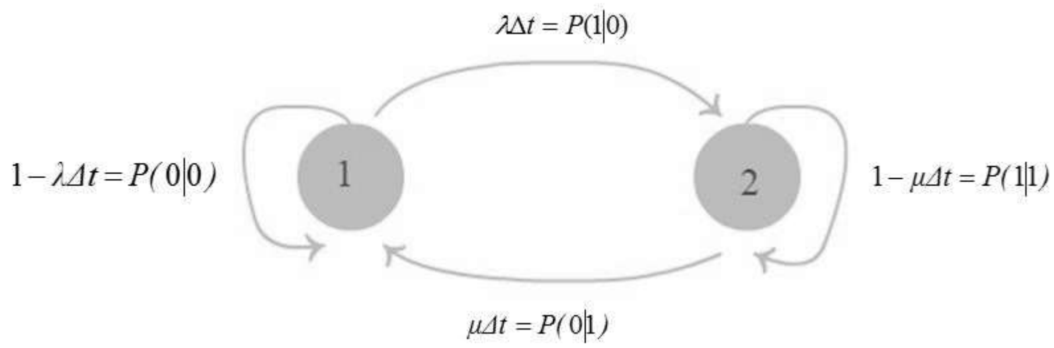

Fig. 1. Markov state graph: 1 - working condition; 2 - inoperative condition

The probability of a system failure is the probability that $x(t+\Delta t)=1$. This probability, in turn, can be expressed in terms of two possible states $x(t)$ and corresponding transitions to the state $x(t+\Delta t)=1$:

$$Q(t+\Delta t)=\Pr\left[x(t+\Delta t)=1\right]=P(1|0)\cdot\Pr\left[x(t)=0\right]+P(1|1)\cdot\Pr\left[x(t)=1\right]=$$
$$=\lambda\cdot\Delta t\left[1-Q(t)\right]+(1-\mu\cdot\Delta t)\cdot Q(t).$$

The last equation can be rewritten as:

$$Q(t+\Delta t)=\lambda\cdot\Delta t-\lambda\cdot\Delta t\cdot Q(t)+Q(t)-\mu\cdot\Delta t\cdot Q(t).$$

From where do we find:

$$\frac{dQ}{dt}=-(\lambda+\mu)Q(t)+\lambda, \tag{2}$$

with the following initial conditions $Q(0)=0$.

Fig. 2 reflects the behavior of the system (the introduction into the system of additional elements in excess of the minimum required number), consisting of elements A and B [11]. Each rectangle in this figure reproduces one state of such a system. The leftmost cell in each of the rectangles is intended to indicate the spare component, the middle cell is to indicate the main component, and the rightmost cell is to indicate the component currently under repair. Therefore, rectangle 1 shows the state in which component B is the backup and component A is the principal. Similarly, rectangle 4 reproduces a state in which component B is the principal and component A is in repair. Possible state transitions in the figure are reproduced by arrows. Transitions from state 1 to state 3 and from state 2 to state 4 are characteristic only for partially loaded and loaded reservations; these transitions are not available for unloaded reservations.

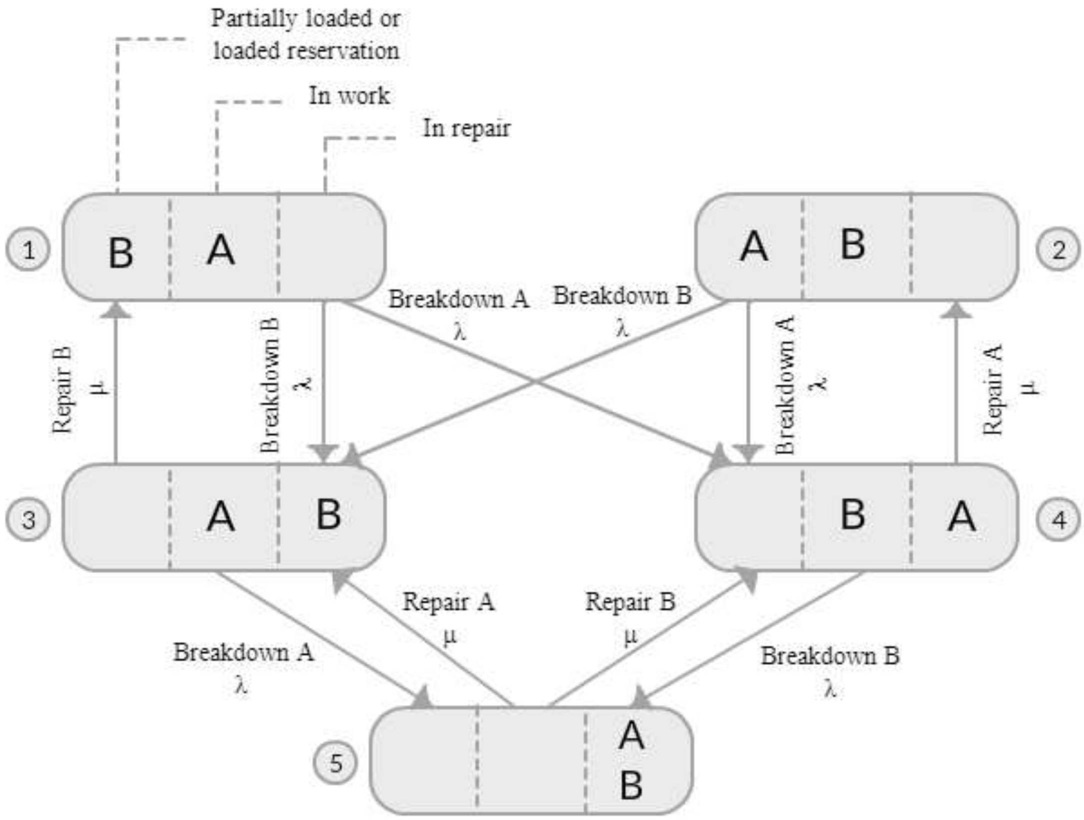

Fig. 2. Markov transition graph when booking

In the case of partially loaded or loaded redundancy, it is assumed that the failure of the reserve components is characterized by a constant intensity $\bar{\lambda}$. In the case of a loaded reservation $\bar{\lambda}$, it is considered equal λ to the failure rate of the main component. With unloaded redundancy $\bar{\lambda}$ equals zero.

Special cases of partially loaded redundancy ($0 < \bar{\lambda} < \lambda$) are unloaded redundancy ($\bar{\lambda} = 0$) and unloaded redundancy ($\bar{\lambda} = \lambda$). The recovery rate of all components in the system is the same and equal μ. For all types of redundancy considered above, the system is considered to have failed if it went to state 5.

Denote by $P_i(t)$ the probability that the system is in a state i at time t. The derivative of this probability is as follows:

$P'(t) =$ state transition rate i - state transition rate $i = \sum_j$ (intensity of transition from state j to state i) $\times$ probability of a state occurrence $j - \sum_j$ (intensity of transition from state i to state j) $\times$ state probability i.

The use of the expression given for the system under consideration makes it possible to construct the following system of differential equations:

$$\begin{aligned}
P_1' &= -(\lambda + \bar{\lambda})P_1(t) + \mu P_3(t) \\
P_2' &= -(\lambda + \bar{\lambda})P_2(t) + \mu P_4(t) \\
P_3' &= \bar{\lambda} P_1(t) + \lambda P_2(t) - (\lambda + \mu)P_3(t) + \mu P_5(t) \\
P_4' &= \bar{\lambda} P_2(t) + \lambda P_1(t) + (\lambda + \mu)P_4(t) + \mu P_5(t) \\
P_5' &= \lambda P_3(t) + \lambda P_4(t) - 2\mu P_5(t)
\end{aligned} \tag{3}$$

The first equation in (3) reflects the fact that the intensity of the flow directed from state 3 to state 1 is equal μ, and the intensities of flows directed from state 1 to states 3 and 4, respectively, λ and $\bar{\lambda}$. Similarly receive other equations.

Suppose that the system under consideration at time zero is in state 1, that is, at time zero both components are operational, with component B in reserve and component A in operation. On the basis of this assumption, we can thus determine the initial conditions for differential equations (3):

$$P_1(0) = 1; P_i(0) = 0, \qquad i = 2, \ldots, 5 \tag{4}$$

Adding the first equation of system (3) with the second and third equation with the fourth, we obtain

$$\begin{aligned}
\frac{dP_0}{dt} &= -(\lambda + \bar{\lambda})P_0 + \mu P_1; \\
\frac{dP_1}{dt} &= (\lambda + \bar{\lambda})P_0 - (\lambda + \mu)P_1 + 2\mu P_2; \\
\frac{dP_2}{dt} &= \lambda P_1 - 2\mu P_2
\end{aligned} \tag{5}$$

with initial conditions $P_0(0) = 1; P_1(0) = P_2(0) = 0;$

where $P_0 = P_1(t) + P_2(t); \quad P_1 = P_3(t) + P_4(t); \quad P_2 = P_5(t).$

The system of differential equations (5) describes a system whose transition graph contains three states - (0), (1), and (2) (Fig. 3). The intensity of the transition stream coming out of state (0) is equal $\lambda + \bar{\lambda}$, and the intensity of the input stream is μ.

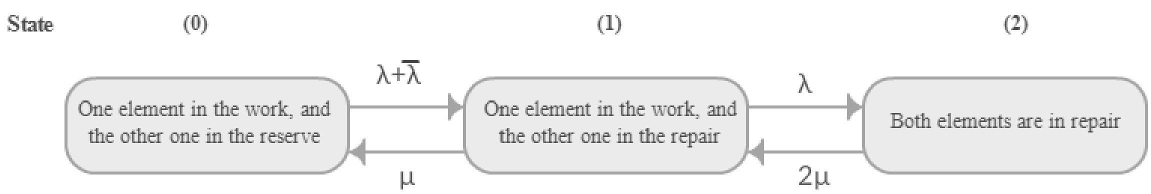

Fig. 3. A simplified Markov transition graph for system backup

In Fig. 4 shows the dependence of the probability of failure of elements $\{A, B\}$ ($Qr(t) = \Pr(A \cap B)$) on time and numerically equal to the probability that both components A and B are in repair ($Qr-$ curve partially loaded redundancy at values $\lambda = 10^{-3} year^{-1}$;

$\bar{\lambda} = 0{,}5 \cdot 10^{-3} year^{-1}$; $\mu = 10^{-2} year^{-1}$ curve $QrN-$ loaded state at values $\bar{\lambda} = 10^{-3} year^{-1}$; $\mu = 10^{-2} year^{-1}$; curve $QrNN-$ unloaded redundancy at values $\lambda = 10^{-3} year^{-1}$; $\bar{\lambda} = 0$; $\mu = 10^{-2} year^{-1}$). Analysis of the results of the numerical solution of system (5) ($Qr(t) = P_2(t)$) shows that the probability of failures is higher in the case of loaded redundancy than in the case of partially loaded and higher in the case of unloaded redundancy.

We calculate the probability of failure of the system as a whole. We accept for the pumps of the cooling device of a failure rate $\lambda = 10^{-3} year^{-1}$; $\bar{\lambda} = 0{,}5 \cdot 10^{-3} year^{-1}$; $\mu = 10^{-2} year^{-1}$ (partially loaded redundancy), and for pumps providing steam circulation in the gas purification column $\lambda = 10^{-3} year^{-1}$; $\bar{\lambda} = 0$; $\mu = 10^{-2} year^{-1}$ (unloaded redundancy). The failure and recovery rates for compressor C, water pump E and filter H are as follows $\lambda^* = 10^{-4} year^{-1}, \mu^* = 10^{-2} year^{-1}$

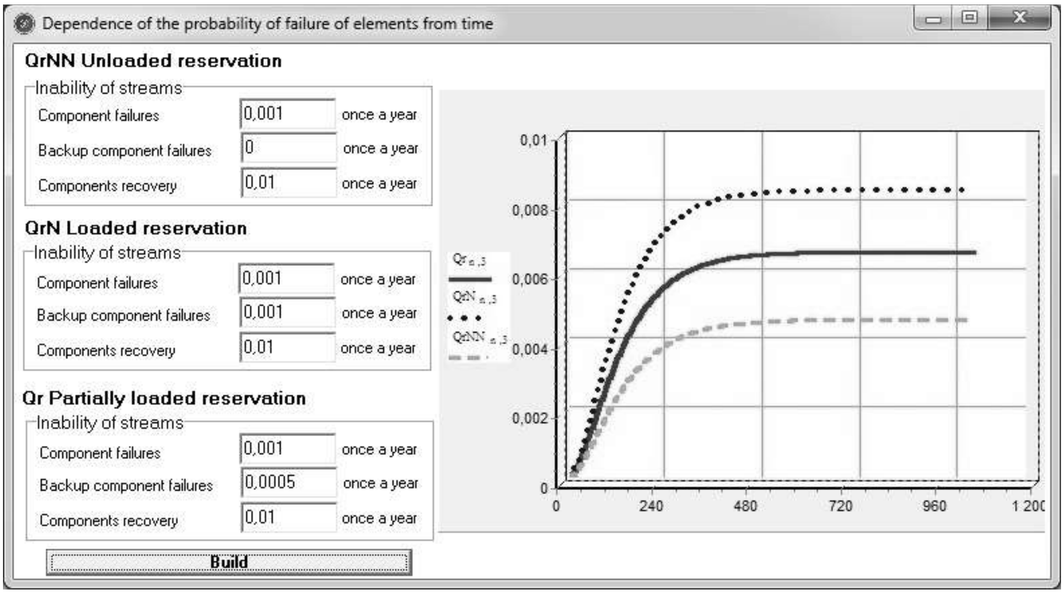

Fig. 4. Dependencies of the probabilities of failure of elements A and B on time

The numerical solution of problem (5) gives the following values of failure probabilities (Table 1):

Table 1. System failure probabilities

t	100	500	1000
$Qr(t)$	0,0028024	0,0064155	0,006479
$QrNN(t)$	0,0018982	0,0044707	0,0045245
$Q(t)$	0,0062948	0,0098375	0,0099006
$Q_s(t)_{\max}$	0,023585	0,040399	0,40705
$Q_s(t)_{\min}$	0,023372	0,039758	0,040055

The failure rates for compressor C, water pump E and filter H can be calculated by the equation:

$$Q(t) = \Pr(C) = \Pr(E) = \Pr(H) = \frac{\lambda^*}{\lambda^* + \mu^*} \cdot \left\{1 - \exp\left[-(\lambda^* + \mu^*) \cdot t\right]\right\}. \tag{6}$$

Dependence (6) is a solution of the differential equation (2)

$$\frac{dQ}{dt} = -(\lambda + \mu) \cdot Q(t) + \lambda, \tag{7}$$

which describes a Markov graph of the states of the inability and inability of a component at constant values of the intensities of failures and recoveries [12].

Applying the Laplace transform, we have:

$$pQ(p) = -(\lambda + \mu)Q(p) + \frac{\lambda}{p}, \qquad Q(P)\left[p + (\lambda + \mu)\right] = \frac{\lambda}{p}.$$

where

$$Q(p)=\frac{\lambda}{p\left[p+(\lambda+\mu)\right]}=\frac{A}{p}+\frac{B}{p+(\lambda+\mu)},$$

where

$$\lambda=Ap+A(\lambda+\mu)+Bp\;.$$

Solving the system

$$\begin{cases}A+B=0\\ A(\lambda+\mu)=\lambda\end{cases}$$

we find

$$A=\frac{\lambda}{\lambda+\mu};B=-\frac{\lambda}{\lambda+\mu}$$

Finally we find

$$Q(p)=\frac{\frac{\lambda}{\lambda+\mu}}{p}-\frac{\frac{\lambda}{\lambda+\mu}}{p+(\lambda+\mu)}\overset{\bullet}{\underset{\bullet}{\rightarrow}}\frac{\lambda}{\lambda+\mu}-\frac{\lambda}{\lambda+\mu}\exp\left[-(\lambda+\mu)t\right]. \quad (8)$$

The probability of system failure is generally calculated as:
upper limit for rejection

$$Q_s(t)_{\max}=3\cdot Q(t)+Qr(t)+QrNN(t), \quad (9)$$

the lower limit of failure

$$Q_s(t)_{\min}=Q_s(t)_{\max}-3\cdot Q(t)^2-3\cdot Q(t)\cdot Qr(t)-3\cdot Q(t)\cdot QrNN(t)-Qr(t)\cdot QrNN(t). \quad (10)$$

Let us now consider a system where pumps of a cooling device are switched on in the scheme "2 of 3". Suppose that the failure rate of each of the cooling unit pumps is equal to λ when the corresponding pump is in operation $\bar{\lambda}$ and when it is in reserve.

The transition graph for this system is shown in Fig. 5. Condition (0) corresponds to the situation when two pumps are in operation and one is in reserve. State (0) corresponds to three substations (1, 2, 3), each of which can go to state (1), and the intensities of the respective transitions are the same and make up $2\lambda+\bar{\lambda}$. The intensity of the transition from state (0) to state (1) is given by:

$$(2\lambda+\bar{\lambda})P_1+(2\lambda+\bar{\lambda})P_2+(2\lambda+\bar{\lambda})P_3=(2\lambda+\bar{\lambda})(P_1+P_2+P_3)=(2\lambda+\bar{\lambda})P_0$$

This means that the intensity of the transition from state (0) to state (1) is equal to ($2\lambda+\bar{\lambda}$), as shown in Fig. 5. The intensities of other transitions can be determined similarly.

The transition graph shown in Fig. 5, the following system of differential equations [13] corresponds:

$$\begin{aligned}\frac{dP_0}{dt}&=-(2\lambda+\bar{\lambda})P_0+\mu P_1;\\ \frac{dP_1}{dt}&=(2\lambda+\bar{\lambda})P_0-(2\lambda+\mu)P_1+\mu P_2;\\ \frac{dP_2}{dt}&=2\lambda P_1-(\lambda+\mu)P_2+\mu P_3\\ \frac{dP_3}{dt}&=\lambda P_2-\mu P_3\end{aligned} \quad (11)$$

under initial conditions $P_0(0)=1; P_1(0)=P_2(0)=P_3(0)=0.$

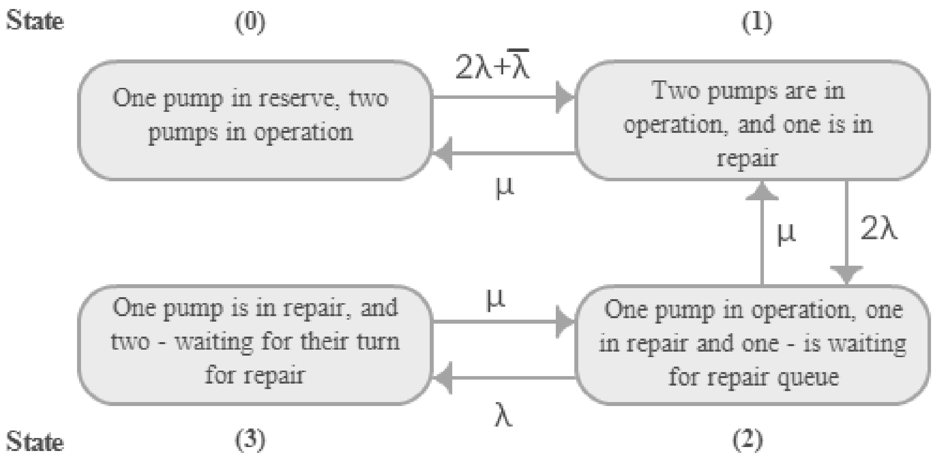

Fig. 5. A simplified Markov transition graph for redundancy reservation according to the scheme "2 of 3"

As a result of solving this system of differential equations, one can determine the probabilities of states.

In order to be able to work, it is necessary that at least two of the three pumps of the cooling unit available are functional. Thus, the parameter value $Q_r(t)$ for the cooling system of the system under consideration, which is equal to the probability that "less than two cooling system pumps are operable", is given by the following expression:

$$Q_r(t) = P_2(t) + P_3(t).$$

The numerical solution of problem (11) gives the following values of failure probabilities (Fig. 6) ($\lambda = 10^{-3}$ *год*$^{-1}$; $\bar{\lambda} = 0,5 \cdot 10^{-3}$; $\mu = 10^{-2}$ *год*$^{-1}$).

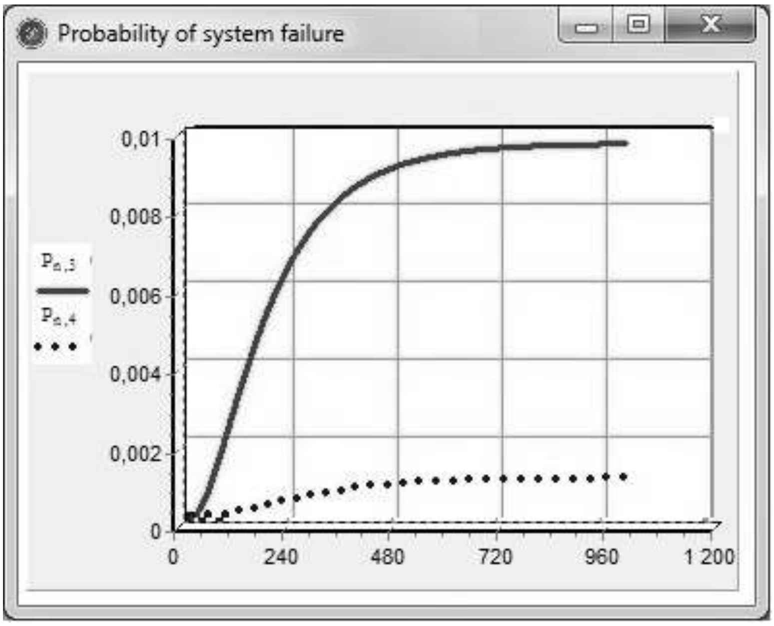

Fig. 6. Probabilities and system solutions (11)

The error tree for the system of cooling and purification of fugitive gas during redundancy reservation according to the scheme "2 of 3" (Fig. 7) has seven minimum failure sections:

$$\{C\},\{E\},\{H\},\{A,B\},\{B,D\},\{D,A\},\{F,G\}.$$

The upper and lower bounds of the system failure probability are:

$$Q_s(t) = \Pr\{C \cup E \cup H \cup [(A \cap D) \cup (B \cap D) \cup (D \cap A)] \cup (F \cap G)\}. \tag{12}$$

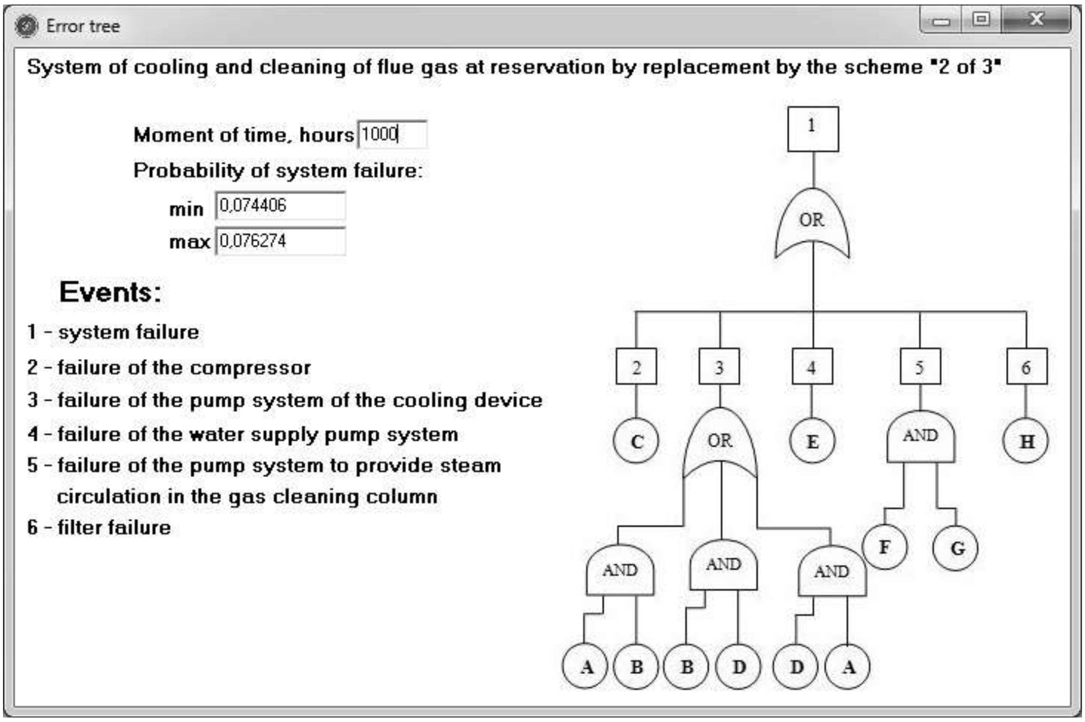

Fig. 7. Error tree for the cooling and purification system of the fugitive gas during redundancy reservation according to the scheme "2 of 3"

Following the above methodology, we calculate the probability of failure of the system (Table 2). The probability of a system failure for a time of 1000 hours lies within $0,0744064 \le Q_s(t) \le 0,076274$.

Table 2. System failure probabilities

t	100	500	1000
$P_2(t)$	0,011429	0,036325	0,038238
$P_3(t)$	0,000357	0,003330	0,003810
$Q_r(t)$	0,011786	0,039655	0,042048
$Q_s(t)_{\max}$	0,032569	0,073638	0,076274
$Q_s(t)_{\min}$	0,032169	0,071868	0,074406

In the general case, substitution redundancy must satisfy the following conditions [14]:

1. The chain contains n identical components.

2. In order to ensure the link's performance, it is necessary that at least m of the link components be operable ($1 \le m \le n$).

3. No more than r link components may be updated at any one time.

The circuit of the scheme "m with n" is described by the following system of differential equations:

$$\frac{dP_0}{dt} = -\lambda_0 P_0 + \mu_1 P_1;$$
$$\frac{dP_k}{dt} = -\lambda_{k-1} P_{k-1} - (\lambda_k + \mu_k) P_k + \mu_{k+1} P_{k+1}; \quad (13)$$
$$\frac{dP_n}{dt} = -\lambda_{n-1} P_{n-1} - \mu_n P_n,$$

where

$$\lambda_k = m\lambda + (n-m-k)\cdot\bar{\lambda}, \quad 0 \le k \le n-m;$$

$$\lambda_k = (n-k)\cdot\lambda, \quad n-m+1 \le k \le n-1;$$

$$\mu_k = \min\{r,k\}\cdot\mu, \quad 1 \le k \le n.$$

The value $Q_r(t)$ is calculated by the expression:

$$Q_r(t) = P_{n-m+1}(t) + ... + P_n(t).$$

The system (5) discussed above is a separate case of system (13) at $n = 2, m = 1, r = 2,$ and system (11) a case of system (13) at $n = 3, m = 2, r = 1.$

Conclusions.

1. No complex system can have absolute security. However, society cannot allow the possibility of serious accidents when operating such systems. Therefore, one of the main tasks of science is the justification of quantitative security requirements and the creation of methods for calculating security systems with risk.

2. The Markov Process Model is an adequate method for analyzing the fault tolerance of systems. This method works well with bounce trees - a well known tool for reliability.

3. The obtained calculations of the approximate probability of failure of the system allow to analyze the failures of technical systems in order to improve the reliability of their functioning.

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