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SOFTWARE SELECTION ON BASE OF FUZZY AHP METHOD

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ABSTRACT

The article is devoted to the problem of multi-criteria decision making. As application problem is used the software selection problem. The analysis of existing methods for solving this problem is given. As a method for solving this problem, the most popular fuzzy AHP method (Analytic Hierarchy Process) is proposed. This method use original algorithm for pairwise comparison of criteria and alternatives. The issues of practical implementation of this method are discussed in details. The results of the solution test problem at all stages are presented.

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1. Introduction. The problem of multi-criteria decision making – (MCDM) is one of the actual problem in the theory of decision making [1-2]. From a mathematical point of view, it belongs to the class of vector optimization problems. The criteria can be divided into two groups: the criteria for which the maximum value is optimal and the criteria for which the minimum value is optimal. MCDM problems can be solved with an accuracy of many non-dominated alternatives or many trade-offs. Obtaining a single solution can only be implemented on the basis of some compromise scheme that reflects the preferences of the decision maker (DM). Methods for solving this problem can be divided into two large groups: methods using the aggregation of all alternatives according to all criteria and the solution of the resulting single criterion problem, the second group is associated with the procedure of pairwise comparisons and stepwise aggregation. The first group includes methods: weighted average sum, weighted average product and their various modifications [3-4], the second group includes - Analytic Hierarchy Process (AHP), Elimination and Choice Translating Reality (ELECTRE), The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Preference Ranking Organization Method (PROMETHEE) [5-15]. This paper discusses the fuzzy AHP method.

The method of AHP (Analytic hierarchy process) was proposed in the early 80's one of the greatest authorities in the field of operations research professor at Pittsburgh University (USA) Thomas Saaty.

An important part of all decision making algorithms is the process of determining the weighting coefficients of the criteria. In many methods, these coefficients are assigned by an expert, which does not always lead to adequate values. A main feature of AHP method is original procedure for calculating weighting coefficients criteria and alternatives on the basis of a single procedure paired comparisons.

Another feature of the AHP method is the consistent use of structural approach to the problem of decision making.

The decision making problem is presented as a hierarchical structure - goal-criteria-alternatives. At present, AHP is the most popular method for solving multi-criteria decision making problems [3]. AHP's popularity is largely due to the use of intuitive technology paired comparisons and procedures weighted average. Let's consider the description of AHP.

2. Description of the method

Suppose MCDM is given in the form of a matrix of outcomes (alternatives - criteria) m - number of criteria n - number of alternatives.

	C ₁	C ₂	C ₃		C _m
A ₁					
A ₂					
A ₃					
A _n					

Fig. 1. MCDM problem representation

AHP is implemented in the form of a sequential multi-stage procedure. At first stage DM builds a matrix of paired comparisons of criteria, for identifying the rank criterion and accordingly the weight criteria for calculating global assessment. The ranks of each criterion are calculated on base of the preference scale and the corresponding indices. Each pair can be defined on a linguistic scale, mapped to an interval (1-9).

T.Saaty proposed an original scale for evaluating paired comparisons

- 1 - the criteria are of equal importance,
- 3 - one criterion is somewhat more important than the other,
- 5 - one criterion is significantly more important than another,
- 7 - one criterion is undeniably more important than another
- 9 - one criterion is absolutely more important than another.

The matrix of pairwise comparisons of criteria ($m \times m$) is presented in Fig. 2.

	C ₁	C ₂		C _m
C ₁	1	C ₁ /C ₂			C ₁ /C _m
C ₂	C ₂ /C ₁	1	C ₂ /C _m
C ₃	C ₃ /C ₁		1		
...	1
C _m	C _m /C ₁	C _m /C ₂	1

Fig. 2. Pairwise comparison of criterion matrix

Here C_i/C_j , preference index of C_i criterion over C_j

In fuzzy AHP (FAHP) are used fuzzy numbers. In this article trapezoidal fuzzy numbers (TFN) are used.

Definition: Trapezoidal fuzzy number

Trapezoidal fuzzy number, $\tilde{A} = (a, b, c, d)$, has following membership function:

$$\mu_{\tilde{A}}(x) = \left\{ \begin{array}{ll} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{x-c}{d-c} & c \leq x \leq d \\ 0 & x > d \end{array} \right\}$$

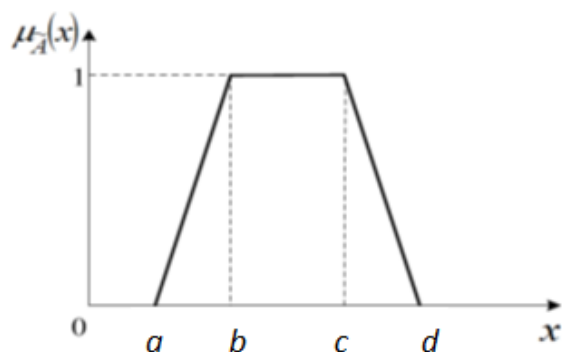


Fig. 3. Trapezoidal fuzzy number

Let's consider basic mathematical operations with two TFT numbers: \tilde{A}_1 and \tilde{A}_2 :

$$\tilde{A}_1 = (a_1, b_1, c_1, d_1) \quad \tilde{A}_2 = (a_2, b_2, c_2, d_2)$$

Addition

$$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

Subtraction

$$\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2)$$

Scalar multiplication $\lambda \tilde{A} = \begin{cases} (\lambda a, \lambda b, \lambda c, \lambda d) & \lambda > 0 \\ (\lambda d, \lambda c, \lambda b, \lambda a) & \lambda < 0 \end{cases}$

Division

$$\tilde{A}_1 \oslash \tilde{A}_2 = (a_1/d_2, b_1/b_2, c_1/c_2, d_1/a_2)$$

Inverse $\tilde{A}^{-1} = \frac{1}{\tilde{A}} \approx (1/d, 1/c, 1/b, 1/a)$

Ranking function

For ranking alternatives is used the function proposed by R.Yager:

$$R(\tilde{A}) = \frac{1}{2} (a + d - \frac{4b}{5} + \frac{2c}{3})$$

Let's \tilde{A}_i and \tilde{A}_j two TFN numbers ,

- (i) $R(\tilde{A}_i) > R(\tilde{A}_j)$ then $\tilde{A}_i > \tilde{A}_j$
- (ii) $R(\tilde{A}_i) < R(\tilde{A}_j)$ then $\tilde{A}_i < \tilde{A}_j$
- (iii) $R(\tilde{A}_i) = R(\tilde{A}_j)$ then $\tilde{A}_i = \tilde{A}_j$

Paiwise preferences are defined as follows:

$1=(1,1,1,1)$ – two criteria are of absolutely equal importance

$\tilde{1} = (0,1,1.5,2)$ –one criterion is little more important than the other,

$\tilde{3} = (2,2.5,3.5,4)$ - one criterion is somewhat more important than the other,

$\tilde{5} = (4,4.5,5.5,6)$ - one criterion is significantly more important than another,

$\tilde{7} = (6,6.5,7.5,8)$ - one criterion is undeniably more important than another,

$\tilde{9} = (8,8.5,9,9)$ - one criterion is absolutely more important than another.

It is important to note that if the preference C_i over C_j is 5, then the preference C_j over C_i is 1/5, this relationship is called inverse symmetry. Logical transitivity must also be performed. If both conditions are met then the matrix is called consistency, otherwise the inconsistency.

Fuzzy inverse symmetric preference will be

$$\tilde{1}^{-1} = (1/2, 1/1, 1, 1)$$

$$\tilde{3}^{-1} = (1/4, 1/3.5, 1/2.5, 1/2)$$

$$\tilde{5}^{-1} = (1/6, 1/5.5, 1/4.5, 1/4)$$

$$\tilde{7}^{-1} = (1/8, 1/7.5, 1/6.5, 1/6)$$

$$\tilde{9}^{-1} = (1/9, 1/9, 1/8.5, 1/8)$$

If is needed intermedia values can be used:

$$\tilde{2} = (1,1.5,2.5,3)$$

$$\tilde{4} = (3,3.5,4.5,5)$$

$$\tilde{6} = (5,5.5,6.5,7)$$

$$\tilde{8} = (7,7.5,8.5,9)$$

The corresponding inversy symmetric numbers

$$\tilde{2}^{-1} = (1/3,1/2.5, 1/1.5, 1/1)$$

$$\tilde{4}^{-1} = (1/5,1/4.5, 1/3.5, 1/3)$$

$$\tilde{6}^{-1} = (1/7,1/6.5, 1/5.5, 1/5)$$

$$\tilde{8}^{-1} = (1/9,1/8.5, 1/7.5, 1/7)$$

T. Saaty proposed to use as an aggregator average geometric mean

$$\tilde{G}_i = \sqrt[m]{\prod_{i=1}^m \tilde{X}_i} , \tag{1}$$

where: $\tilde{X}_i = \tilde{C}_i/\tilde{C}_j$ fuzzy preference index of C_i over C_j

Next for any row of pairwise preferences matrix we calculate aggregated index and weights of criterias by formulas:

$$\tilde{S} = \sum_{i=1}^m \tilde{G}_i, \quad \tilde{w}_i = \frac{\tilde{G}_i}{\tilde{S}} , \tag{2}$$

As result we have vector of fuzzy criterias weights :

$$\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \tilde{w}_3 \dots \tilde{w}_m)$$

At second stage for any criterion C_i are determined pairwise comparison matrix (Fig 4.)

	A₁	A₂	A_n
A₁	1	\tilde{A}_1/\tilde{A}_2			\tilde{A}_1/\tilde{A}_n
A₂	\tilde{A}_2/\tilde{A}_1	1		...	\tilde{A}_2/\tilde{A}_n
A₃	\tilde{A}_3/\tilde{A}_1		1		
...	1	...
A_n	\tilde{A}_n/\tilde{A}_1	\tilde{A}_n/\tilde{A}_2		...	1

Fig. 4. Matrix of pairwise comparison of alternatives

For any matrix are calculated normalised preference indexes (2)

On base of these indexes the consolidated preference indexes matrix is build, (Fig. 5.)

	C₁	C₂	C₃		C_m
A₁	\tilde{X}_{11}	\tilde{X}_{12}	\tilde{X}_{13}		\tilde{X}_{1m}
A₂	\tilde{X}_{21}	\tilde{X}_{22}	\tilde{X}_{23}		\tilde{X}_{2m}
A₃					
A_n	\tilde{X}_{n1}	\tilde{X}_{n2}	\tilde{X}_{n3}		\tilde{X}_{nm}

Fig. 5. The consolidated matrix of preferences indexes of all alternatives

At third stage on base of criterion weight for any alternative are calculated global preference indexes (3):

$$\begin{aligned} \sum_{i=1}^m w_i X_{1i} &= B_1 \\ \sum_{i=1}^m w_i X_{2i} &= B_2 \\ \sum_{i=1}^m w_i X_{3i} &= B_n \end{aligned} \tag{3}$$

At last stage on base of ranking function is determined the alternative with maximum of global preference index.

3. Practice problem solving

As practice problem is considered software selection problem [13-14]. Main criteria are:

C₁- functionality,

C₂- price,

C₃- usability.

C₄– reliability

and four alternatives are proposed.

All calculation were implemented in Ms Excel (Fig. 6).

The screenshot shows an Excel spreadsheet with columns A through Q and rows 1 through 29. It contains various numerical data, matrices, and formulas used for FAHP calculations. Key elements include:

- Columns J, K, L, M: A 4x4 pairwise comparison matrix.
- Columns N, O: Fuzzy preference indices for each criterion (e.g., $\tilde{I} = (0, 1, 1.5, 2)$).
- Columns P, Q: Inverse fuzzy preference indices (e.g., $\tilde{I}^{-1} = (1/2, 1/1, 1/1, 1)$).
- Columns A, B, C, D, E: Final ranking results for four alternatives (A1, A2, A3, A4) across four criteria (C1, C2, C3, C4).

Fig. 6. FAHP computation model in Ms Excel

According to FAHP method for 4 criteria were determined pairwise comparison matrix

	C ₁	C ₂	C ₃	C ₄
C ₁	1	$\tilde{3}$	$\tilde{3}$	$\tilde{2}$
C ₂	$1/\tilde{3}$	1	$1/\tilde{2}$	$1/\tilde{2}$
C ₃	$1/\tilde{3}$	$\tilde{2}$	1	$\tilde{2}$
C ₄	$1/\tilde{2}$	$\tilde{2}$	$1/\tilde{2}$	1

At next step on base of formulas (1) and (2) are calculated weight coefficients of all criteria:

$W_1 = (0.23, 0.33, 0.62, 0.85)$ $W_2 = (0.07, 0.09, 0.17, 0.27)$

$W_3 = (0.11, 0.17, 0.33, 0.47)$ $W_4 = (0.09, 0.13, 0.27, 0.42)$

For any criterion we determine fuzzy pairwise comparison matrix and preference indexes

C₁ criterion

	A ₁	A ₂	A ₃	A ₄	Preference index
A ₁	1	$1/\tilde{2}$	$1/\tilde{3}$	$\tilde{2}$	(0.09, 0.13, 0.26, 0.40)
A ₂	$\tilde{2}$	1	$\tilde{2}$	$1/\tilde{2}$	(0.13, 0.20, 0.42, 0.62)
A ₃	$\tilde{3}$	$1/\tilde{2}$	1	$\tilde{2}$	(0.15, 0.23, 0.45, 0.67)
A ₄	$1/\tilde{2}$	$\tilde{2}$	$1/\tilde{2}$	1	(0.10, 0.14, 0.30, 0.47)

C₂ criterion

	A ₁	A ₂	A ₃	A ₄	Preference index
A ₁	1	$1/\sqrt{2}$	$\sqrt{3}$	$1/\sqrt{2}$	(0.10, 0.14, 0.24, 0.33)
A ₂	$\sqrt{5}$	1	$1/\sqrt{3}$	$\sqrt{2}$	(0.18, 0.25, 0.43, 0.58)
A ₃	$1/\sqrt{3}$	$\sqrt{3}$	1	$\sqrt{2}$	(0.15, 0.21, 0.39, 0.52)
A ₄	$\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$	1	(0.10, 0.15, 0.29, 0.44)

C₃ criterion

	A ₁	A ₂	A ₃	A ₄	Preference index
A ₁	1	$1/\sqrt{3}$	$\sqrt{2}$	$\sqrt{5}$	(0.11, 0.15, 0.27, 0.38)
A ₂	$\sqrt{3}$	1	$\sqrt{2}$	$\sqrt{2}$	(0.17, 0.26, 0.50, 0.70)
A ₃	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$1/\sqrt{2}$	(0.11, 0.15, 0.29, 0.48)
A ₄	$1/\sqrt{5}$	$1/\sqrt{2}$	$\sqrt{2}$	1	(0.12, 0.17, 0.31, 0.45)

C₄ criterion

	A ₁	A ₂	A ₃	A ₄	Preference index
A ₁	1	$\sqrt{5}$	$\sqrt{3}$	$1/\sqrt{3}$	(0.20, 0.26, 0.42, 0.53)
A ₂	$1/\sqrt{5}$	1	$\sqrt{5}$	$\sqrt{2}$	(0.16, 0.21, 0.33, 0.42)
A ₃	$1/\sqrt{3}$	$1/\sqrt{5}$	1	$1/\sqrt{3}$	(0.05, 0.07, 0.11, 0.14)
A ₄	$\sqrt{3}$	$1/\sqrt{2}$	$\sqrt{3}$	1	(0.18, 0.25, 0.42, 0.57)

We consolidate all alternative preference indexes in one matrix.

	C ₁	C ₂	C ₃	C ₄
A ₁	(0.09, 0.13, 0.26, 0.40)	(0.10, 0.14, 0.24, 0.33)	(0.11, 0.15, 0.27, 0.38)	(0.20, 0.26, 0.42, 0.53)
A ₂	(0.13, 0.20, 0.42, 0.62)	(0.18, 0.25, 0.43, 0.58)	(0.17, 0.26, 0.50, 0.70)	(0.16, 0.21, 0.33, 0.42)
A ₃	(0.15, 0.23, 0.45, 0.67)	(0.15, 0.21, 0.39, 0.52)	(0.11, 0.15, 0.29, 0.48)	(0.05, 0.07, 0.11, 0.14)
A ₄	(0.10, 0.14, 0.30, 0.47)	(0.10, 0.15, 0.29, 0.44)	(0.12, 0.17, 0.31, 0.45)	(0.18, 0.25, 0.42, 0.57)

For any criterion on base of formula (3) is calculated global preference indexes of all alternatives:

$$B_1 = (0.06, 0.12, 0.41, 0.83)$$

$$B_2 = (0.07, 0.16, 0.59, 1.19)$$

$$B_3 = (0.06, 0.13, 0.47, 0.99)$$

$$B_4 = (0.06, 0.12, 0.45, 0.97)$$

For any alternative is calculated ranking function:

$$R(A_1) = 0.533554102,$$

$$R(A_2) = \mathbf{0.76396875},$$

$$R(A_3) = 0.634708217,$$

$$R(A_4) = 0.618137562$$

Alternative A₂ have maximum value of global preference index **0.763968**, so A₂ alternative is optimal.

Conclusions. The article is devoted to the problem of multi-criteria decision making for software selection. The analysis of existing methods for solving this problem is given. The fuzzy AHP method is used for solving this problem. The issues of practical implementation of this method are discussed in details.

As test, the problem of software selection problem with 4 criteria and 4 alternatives is considered. The results of the solution at all stages are presented.

REFERENCES

1. Hwang, CL Yoon, K. Multiple Attribute Decision Making: Methods and Applications. New York: Springer- Verlag, 1981
2. V. Belton and T. Stewart, Multiple criteria decision analysis: an integrated approach. Springer Science & Business Media, 2002.
3. A. Mardani, A. Jusoh, Khalil MD Nor, Z. Khalifah, N. Zakwan, A. Valipour Multiple criteria decision-making techniques and their applications - a review of the literature from 2000 to 2014, ISSN: 1331-677X (Print), 2015
4. Chakraborty, S., & Zavadskas, EK Applications of WASPAS method in manufacturing decision making. Informatica, 25 (1), 1-20, 2014
5. Boucher, TO, & MacStravic, E. L. Multi attribute evaluation within a present value framework and its relation to the analytic hierarchy process. The Engineering Economist, 37 (1), 1-32, 1991
6. Taha, RA, & Daim, T. Multi-criteria applications in renewable energy analysis, a literature review. In Research and Technology Management in the Electricity Industry (pp. 17-30). Springer London, 2013
7. Wu, HY, Chen, JK, Chen, IS, & Zhuo, HH Ranking universities based on performance evaluation by a hybrid MCDM model. Measurement, 45 (5), 856-880., 2012
8. Beccali, M., Cellura, M., & Ardente, D. Decision making in energy planning: the ELECTRE multicriteria analysis approach compared to a fuzzy-sets methodology. Energy Conversion and Management, 39 (16-18), 1869-1881, 1998
9. Rogers, M., & Bruen, M. Using ELECTRE III to choose route for Dublin port motorway. Journal of Transportation Engineering, 126 (4), 313-323, 2002
10. Srdjevic, B., & Medeiros, YDP Fuzzy AHP assessment of water management plans. Water Resources Management, 22 (7), 877-894, 2008
11. Meixner, O. Fuzzy AHP group decision analysis and its application for the evaluation of energy sources. In Proceedings of the 10th International Symposium on the Analytic Hierarchy / Network Process, Pittsburgh, PA, USA (Vol. 29), 2009
12. Srichet ta, P., Thurachon Applying fuzzy analytic hierarchy process to evaluate and select product of notebook computers. International Journal of Modeling and Optimization, 2 (2), 168, 2012
13. Azadeh A. A robust decision-making methodology for evaluation and selection of simulation software package / A. Azadeh, SN Shirkouhi, K. Rezaie // International Journal of Advanced Manufacturing Technology. - 2010. - Vol. 47. - P. 381–393.
14. Karaarslan N. An application for modular capability-based ERP software selection using AHP method / N. Karaarslan, E. Gundogar // International Journal of Advanced Manufacturing Technology. – 2009. - Vol. 42. - P. 1025-1033.
15. Smirlis YG Data envelopment analysis models to support the selection of vehicle routing software for city logistics operations / YG Smirlis, V. Zeimpekis, G. Kaimakamis // Operational Research. 2012. –Vol. 12. - P. 399-420.