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INFLUENCE OF WALL SCATTERING EFFECT ON ELECTRONS GAS DYNAMICS PARAMETERS IN ELECTRIC PROPULSION THRUSTERS WITH CLOSED ELECTRON DRIFT

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ABSTRACT

The analysis is represented of some works devoted to the mathematical modeling of processes in plasma-ion thrusters and Hall effect thrusters. It is shown that the common in these works is the use of approximate forms of the equations of gas dynamics, which are applicable to the description of relatively dense gases, but not to analyze the processes in the rarefied plasma of electric propulsion thrusters. As a result, the above mathematical models do not represent the processes that are significantly responsible for the values of the thruster operating parameters.
Authors try to partially correct this drawback by insertion into the initial approximate forms of the equations written for a point in the plasma volume, the parameters that actually represent the boundary effects and should be written not in the equations of gas dynamics themselves, but in the boundary conditions for these equations.
The most complete forms of the necessary equations are given in this paper. It is shown that it is necessary to take into account electrons thermal conductivity as well as at least one (radial-azimuth) component of viscosity tensor to describe the "wall scattering" effect.
It is concluded that the most productive approach in mathematical modeling is to write the most complete forms of equations with their subsequent simplification – removing the terms responsible for the processes recognized on the basis of primary numerical estimates as such, which can be neglected.

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Introduction. Plasma-ion thruster with radial magnetic field and Hall effect thruster relate to electrostatic electric propulsion thrusters with almost axial electric field for ions acceleration and almost radial magnetic field, which is used to prevent the extreme axial electrons current. In the absence of other factors these fields combination would lead to absolutely closed electrons drift without displacement in axial direction.

The most initial attempts to explain the axial electrons current by electron-atom and electron-ion collisions inside the volume had given the value of this current much less than real existing in the thrusters. The main effect of the axial flow of electrons was named the lost of their rotation moment because of non-mirror reflection from a potential barrier in a Langmuir layer near the surface, which was called "near-wall conductivity" or "wall scattering" [1].

However, at the same time, problems arose with the way of describing this effect using the equations of plasma-dynamics, which led to the conclusion that such a description is impossible and that it is necessary to use empirical data or the "particle in a cell" method.

The purpose of this study is to show that these problems are in fact associated with the use of not the most complete forms of the equations of plasma dynamics, but approximate ones, suitable for describing relatively dense gases in contrast to the rarefied plasma of electric propulsion thrusters.

Materials and Methods. The authors of [1] propose to use empirical data to substitute the relaxation frequency of the electrons momentum ν_m as a result of "wall scattering" into the expression for the electrons mobility coefficient $\mu_{e\perp}$. The axial projection $j_{e\perp}$ of electrons current density (formula (7.4-4) [1, 367]) is proposed to be found as:

$$j_{e\perp} = \mu_{e\perp}\left(e\, n_e E_\perp + \frac{\partial P_e}{\partial x} \right) + \ldots, \tag{1}$$

where e, n_e, P_e – electrons charge, population and pressure; $E_\perp$ – axial projection of electric field tension.

In turn, the electrons energy equation (formula (7.4-22) [1, 370]) is written as:

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e \frac{k T_e}{e} \right) + \nabla \cdot \left(\frac{5}{2} T_e \vec{J}_e \right) = \vec{E} \cdot \vec{J}_e - R - S - P_w, \tag{2}$$

where $\vec{J}_e$, T_e – electrons current density and temperature (in eV); R, S, P_w – the radiative energy loss, the ionization energy loss, and the electrons energy loss to the walls.

At the same time, authors do not give any comments on the method of finding the value P_w.

With the use of Clapeyron equation the expression (2) can be written as:

$$\frac{3}{2}\frac{\partial P_e}{\partial t} + \frac{5}{2}\nabla \cdot \left(\vec{V}_e P_e \right) + e\, n_e \vec{V}_e \cdot \vec{E} = \frac{\delta \varepsilon_e^{(V)}}{\delta t} - P_w, \tag{3}$$

where P_e, $\vec{V}_e$ – electrons pressure and mass flow velocity; $\dfrac{\delta \varepsilon_e^{(V)}}{\delta t}$ – electrons energy density change in collisions (excitation, ionization).

Typical in both the cases is the authors' attempt to include boundary effects ("wall scattering" and the electrons energy flow to the walls) into the plasma dynamics equations written for a point in the plasma volume. The need for such an attempt, in fact, appears because the authors in both cases use an incomplete form of the equations themselves and an incomplete form of some terms.

For example, the complex inside the brackets in the first term of (3) must be energy density:

$$\varepsilon_e^{(V)} = \frac{3}{2} P_e + \frac{m_e n_e V_e^2}{2}. \tag{4}$$

The complex inside the brackets in the second term – electrons energy flow density $\vec{q}_e$:

$$\vec{q}_e = \vec{V}_e \left(\frac{3}{2} P_e + \frac{m_e n_e V_e^2}{2} \right) + \vec{V}_e \cdot \boldsymbol{P}_e + \vec{q}_e^{(cond)}, \tag{5}$$

where $\vec{q}_e^{(cond)}$, $\boldsymbol{P}_e$ – electrons thermal conductivity and pressure tensor [2].

The last parameter is the static part of momentum flow density – the second rank tensor $\boldsymbol{\Pi}_e$, which component:

$$\Pi_e^{(mn)} = m_e n_e V_{em} V_{en} + P_e^{(mn)}, \tag{6}$$

is the flow density of m-projection of momentum into n-direction.

So the pressure tensor represents the momentum flow in the absence of mass flow.

Pressure as scalar is defined only as average value of diagonal components of $\boldsymbol{P}_e$:

$$P_e = \frac{P_e^{(11)} + P_e^{(22)} + P_e^{(33)}}{3}. \tag{7}$$

The component of pressure tensor can be represented as:

$$P_e^{(mn)} = \begin{cases} P_e + \pi_e^{(mn)}, & m = n \\ \pi_e^{(mn)}, & m \neq n \end{cases}, \tag{8}$$

and expression (5) can be transformed as:

$$\vec{q}_e = \vec{V}_e \left(\frac{5}{2} P_e + \frac{m_e n_e V_e^2}{2} \right) + \vec{V}_e \cdot \boldsymbol{\pi}_e + \vec{q}_e^{(cond)}. \tag{9}$$

Thus, the most complete forms of three basic gas dynamics equations set are:

- continuity equation:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \left(n_e \vec{V}_e \right) = \frac{\delta n_e}{\delta t}; \tag{11}$$

- motion equation:

$$\begin{gathered} m_e \left(\frac{\partial}{\partial t} \left(n_e \vec{V}_e \right) + n_e \vec{V}_e \cdot \nabla \vec{V}_e + \vec{V}_e \nabla \cdot \left(n_e \vec{V}_e \right) \right) + \nabla P_e + \\ + \nabla \cdot \boldsymbol{\pi}_e + e n_e \left(\vec{E} + \vec{V}_e \times \vec{B} \right) = \frac{\delta \vec{p}_e^{(V)}}{\delta t} \end{gathered}; \tag{12}$$

- energy equation:

$$\begin{gathered} \frac{\partial}{\partial t} \left(\frac{3}{2} P_e + \frac{m_e n_e V_e^2}{2} \right) + \nabla \cdot \left(\vec{V}_e \left(\frac{5}{2} P_e + \frac{m_e n_e V_e^2}{2} \right) + \vec{V}_e \cdot \boldsymbol{\pi}_e + \vec{q}_e^{(cond)} \right) + \\ + e n_e \vec{V}_e \cdot \vec{E} = \frac{\delta \varepsilon_e^{(V)}}{\delta t} \end{gathered}, \tag{13}$$

where $\dfrac{\delta n_e}{\delta t}$ – electrons population change in collisions (ionization).

Viscosity and thermal conductivity together are named as dissipative corrections.

The neglect of viscosity in the motion equation, as was done in [1], as well as in [3, 4] and both dissipative corrections in the energy equation was the reason for the subsequent artificial insertions.

Results. The "wall scattering" is the boundary effect and should be represented as the boundary condition in a mathematical model of thruster. Of course, the equations of gas dynamics must contain a parameter, for which this boundary condition is formulated. Non-mirror reflection of electrons from the near-wall potential barrier means their return to the plasma with a smaller value of the azimuth projection of the momentum – there is a flow of the azimuth projection of momentum into the radial direction without a mass flow in this direction and is represented in radial-azimuth component of pressure and viscosity tensor:

$$P_e^{(r\varphi)} = \pi_e^{(r\varphi)}. \quad (14)$$

The factors exist in plasma-ion thruster with radial magnetic field and Hall effect thruster, which make the dispersions of all three projections of electrons velocity almost equal to each other. Thus approximate equivalence exists between diagonal components of pressure tensor:

$$P_e^{(\varphi\varphi)} \approx P_e^{(\varphi\varphi)} \approx P_e^{(\varphi\varphi)} \approx P_e, \quad (15)$$

$$\pi_e^{(xx)} \approx 0, \qquad \pi_e^{(rr)} \approx 0, \qquad \pi_e^{(\varphi\varphi)} \approx 0. \quad (16)$$

Due to essentially subsonic electrons flow it is possible to neglect axial-radial and axial-azimuth components of both the tensors:

$$P_e^{(xr)} \approx 0, \qquad P_e^{(x\varphi)} \approx 0, \quad (17)$$

$$\pi_e^{(xr)} \approx 0, \qquad \pi_e^{(x\varphi)} \approx 0. \quad (18)$$

But neglecting the value $P_e^{(r\varphi)} = \pi_e^{(r\varphi)}$ would mean losing of "wall scattering" effect in the description.

As for radial projection of electrons energy flow density it means:

$$q_{er} = V_{er}\left(\frac{5}{2}P_e + \frac{m_e n_e V_e^2}{2}\right) + V_{e\varphi}\pi_e^{(r\varphi)} + q_{er}^{(cond)}, \quad (19)$$

where boundary condition for q_{er} must be obtained as:

$$q_{ew} = \frac{m_e}{2}\int_{v_{r\,max}}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{ew}\left(v_r, v_{||1}, v_{||2}\right)\left(v_r^2 + v_{||1}^2 + +v_{||2}^2\right)v_r d\,v_r d\,v_{||1} d\,v_{||2}, \quad (20)$$

where f_{ew} – electrons velocity distribution function; v_r, $v_{||1}$, $v_{||2}$ – normal and two parallel to the wall electrons velocity projections.

For comparatively dense plasma it is possible to use Maxwell form of f_{ew}. But for plasma-ion and Hall effect thruster it is better to obtain the approximate solution of kinetic equation for electrons.

The boundary condition for the value $\pi_e^{(r\varphi)}$ was obtained in [5] as a result of solving the kinetic problem of the motion of electrons in a Langmuir layer:

$$\pi_{ew}^{(r\varphi)} = \eta_P \frac{v_e}{4} m_e n_e V_{e\varphi}, \quad (21)$$

where v_e – electrons velocity mean absolute value; η_P – electron's motion relaxation factor in "wall scattering".

The will to write the equation not for the total energy, but only for its thermal component is quite natural, but the simple exclusion of the terms quadratic in the velocity from (13), as was done in (2) and (3), is unacceptable. This can be shown with a simple example. Let us imagine the case of a uniform distribution of all parameters in space in the absence of energy losses for ionization, excitation, and on the walls. In this case, equation (3) would have the form:

$$\frac{3}{2}\frac{\partial P_e}{\partial t} + e n_e \vec{V}_e \cdot \vec{E} = 0, \quad (22)$$

which would mean the direct influence of the electric field on the velocity dispersion of electrons – the energy of their thermal movement. But the electric field changes the velocity of each separate electron by the same value, that is, it does not change the difference in electrons velocities and cannot directly affect the pressure.

There is an absolutely correct way to transform the total energy equation into a separate thermal energy equation using the equality:

$$\frac{3}{2}\frac{\partial P_e}{\partial t} = \frac{\partial}{\partial t}\left(\frac{3}{2}P_e + \frac{m_e n_e V_e^2}{2}\right) - m_e \vec{V}_e \cdot \frac{\partial}{\partial t}\left(n_e \vec{V}_e\right) + \frac{m_e V_e^2}{2}\frac{\partial n_e}{\partial t}. \quad (23)$$

Thus, from the energy equation it is necessary to subtract the motion equation, all the terms of which are multiplied by $\vec{V}_e$ (in the dot product), and add the continuity equation, all the terms of which are multiplied by the complex $\frac{m_e V_e^2}{2}$. Finally it will mean:

$$\frac{3}{2}\frac{\partial P_e}{\partial t} + \frac{5}{2}\nabla\cdot\left(\vec{V}_e P_e\right) - \vec{V}_e\cdot\nabla P_e + \\ + \nabla\cdot\left(\vec{V}_e\cdot\boldsymbol{\pi}_e\right) - \vec{V}_e\cdot\left(\nabla\cdot\boldsymbol{\pi}_e\right) + \nabla\cdot\vec{q}_e^{(cond)} = \frac{3}{2}\frac{\delta P_e}{\delta t}, \quad (24)$$

$$\frac{3}{2}\frac{\delta P_e}{\delta t} = \frac{\delta \varepsilon_e^{(V)}}{\delta t} - \frac{\delta \vec{p}_e^{(V)}}{\delta t} + \frac{m_e V_e^2}{2}\frac{\delta n_e}{\delta t}. \quad (25)$$

The right part in (24) as well as the last term in the left part relate to the loss of electrons both total and thermal energy. But the complex $\nabla\cdot\left(\vec{V}_e\cdot\boldsymbol{\pi}_e\right) - \vec{V}_e\cdot\left(\nabla\cdot\boldsymbol{\pi}_e\right)$ in our case is equal to:

$$\nabla\cdot\left(\vec{V}_e\cdot\boldsymbol{\pi}_e\right) - \vec{V}_e\cdot\left(\nabla\cdot\boldsymbol{\pi}_e\right) \approx -\pi_e^{(r\varphi)}\frac{V_{e\varphi}}{r}. \quad (26)$$

The sign of $\pi_e^{(r\varphi)}$ is the same that the sign of $V_{e\varphi}$ – azimuth projection of momentum is transported into radial direction. Minus in the right part of (26) means the part of electrons thermal energy transport from the wall into plasma – "wall scattering" effect means an increase in the dispersion of electrons in velocities, converting part of the rotation energy of electrons into thermal energy.

The terms quadratic in velocity can indeed be neglected in motion equation of electrons. In this case, the axial and azimuth projections of equation (12) in radial magnetic field take the form:

$$m_e\frac{\partial}{\partial t}\left(n_e V_{ex}\right) + \frac{\partial P_e}{\partial x} + e n_e\left(E_x - V_{e\varphi}B\right) = \frac{\delta p_{ex}^{(V)}}{\delta t}, \quad (27)$$

$$m_e\frac{\partial}{\partial t}\left(n_e V_{e\varphi}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\pi_e^{(r\varphi)}\right) + e n_e V_{ex}B = \frac{\delta p_{e\varphi}^{(V)}}{\delta t}, \quad (28)$$

where "wall scattering" is represented by the second term in the left part of (28).

The discussion of the results. It is possible to imagine two approaches to mathematical modeling:

- writing the most complete forms of equations with their following simplification – removing the terms responsible for the processes recognized on the basis of primary numerical estimates as such that can be neglected;

- writing of known approximate forms of equations with their following correction for processes not taken into account in the primary approximate forms.

The first approach is the most clear – someone sees the compound, which can be deleted and understands the reasons of this deletion. The second approach inevitably has the character of speculations and depends on the ability of the researcher to imagine in detail all the circumstances that require additional consideration.

Thus, the first approach is the only acceptable one.

Conclusions. The analysis of existing publications devoted to the mathematical modeling of processes in plasma-ion and Hall thrusters showed a tendency to use approximate forms of equations applicable to relatively dense gases. Attempts to correct these approximate forms for describing the rarefied plasma of electric propulsion thrusters often lead to losses in the description of the processes responsible for the thruster performance, underestimated or overestimated values of many parameters.

The system of equations is presented, built by reasonable simplification of the most complete primary equations.

It is shown that an adequate description requires taking into account the thermal conductivity and at least one off-diagonal component of the viscosity tensor.

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