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APPLICATION OF TOPSIS METHOD WITH TRAPEZOIDAL FUZZY NUMBERS

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ABSTRACT

The article is devoted to the problem of multi-criteria decision making. As application problem is used the equipment selection problem. The analysis of existing methods for solving this problem is given. As a method for solving this problem fuzzy TOPSIS (The Technique for Order of Preference by Similarity to Ideal Solution) is proposed. This method is based on ideal solution approach. The issues of practical implementation of this method are discussed in details. The results of the solution test problem at all stages are presented.

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Introduction. Multi Criteria Decision making – (MCDM) is one of the actual problem in the theory of decision making [1-2]. From a mathematical point of view, it belongs to the class of vector optimization problems. The criteria can be divided into two groups: the criteria for which the maximum value is optimal and the criteria for which the minimum value is optimal. MCDM problems can be solved with an accuracy of many non-dominated alternatives or many trade-offs. Obtaining a single solution can only be implemented on the basis of some compromise scheme that reflects the preferences of the decision maker (DM). Methods for solving this problem can be divided into two large groups: methods using the aggregation of all alternatives according to all criteria and the solution of the resulting single-criterion problem, the second group is associated with the procedure of pairwise comparisons and stepwise aggregation. The first group includes methods: weighted average sum, weighted average product and their various modifications [3-4], the second group includes -Analytical Hierarchy Process (AHP), Elimination and Choice Translating Reality (ELECTRE), The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Preference Ranking Organization Method (PROMETHEE) [5-13]. The work [3] provides information on the popularity of various methods of multi-criteria decision-making. This paper discusses the TOPSIS method.

The TOPSIS method was developed by Hwang and Yoon in 1981. This method was very popular for solving multi-criteria problem under certain conditions. In general the TOPSIS method is based on the approach of ideal solution.

The fuzzy TOPSIS [4-13] method was developed by Chen in 2000 for problem with linguistic uncertainty.

Description of the method.

We consider the problem where decision DM makes decisions in linguistic form.

Consider all stages of fuzzy TOPSIS method:

1. First we define linguistic variables for criterion weight importance and the decisions with fuzzy trapezoidal numbers.

Table 1. Linguistic variables for the importance of criterion weights

Linguistic Variables	Trapezoidal Fuzzy Numbers
Very Low (VL)	(0,0.1,0.2,0.3)
Low (L)	(0.1,0.3,0.45,0.7)
Medium (ML)	(0.4,0.5,0.7,0.8)
High (H)	(0.5,0.6,0.75,0.85)
Very High (VH)	(0.6,0.7,0.8,0.9)

Table 2. Linguistic variables for the decision

Linguistic Variables	Trapezoidal Fuzzy Numbers
Very Poor (VP)	(0,1,2,3)
Poor (P)	(1,3,4,7)
Medium Poor (MP)	(4,5,7,8)
Good (G)	(7,8,9,9.25)
Very Good (VG)	(9, 9.25, 9.5,10)

2. Present the linguistic decisions as the matrix of outcomes (alternatives - criteria)
 n - number of criteria, m - number of alternatives

	C ₁	C ₂	C ₃		C _n
A ₁	\tilde{X}_{11}	\tilde{X}_{12}	\tilde{X}_{13}		\tilde{X}_{1n}
A ₂	\tilde{X}_{21}	\tilde{X}_{22}	\tilde{X}_{23}		\tilde{X}_{2n}
A ₃	\tilde{X}_{31}	\tilde{X}_{32}	\tilde{X}_{33}		\tilde{X}_{3n}
A _m	\tilde{X}_{m1}	\tilde{X}_{m2}	\tilde{X}_{m3}		\tilde{X}_{mn}

Fig. 1. MCDM problem representation

Where $\tilde{X}_{ij} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij})$ is fuzzy trapezoidal representation of linguistic terms.

3. Calculate normalized matrix $\tilde{R} = (r_{ij})$, $i = 1, 2, \dots, m; j = 1, 2, \dots, n$

The normalized fuzzy decision matrix is calculated with the formulas given below, where J and J_1 represent the maximization criteria set, and minimization criteria set respectively.

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*} \right), j \in J$$

$$\tilde{r}_{ij} = \left(\frac{a_j^*}{d_{ij}}, \frac{a_j^*}{c_{ij}}, \frac{a_j^*}{b_{ij}}, \frac{a_j^*}{a_{ij}} \right), j \in J_1$$

$$d_j^* = \max_i d_{ij}, j \in J$$

$$a_j^* = \min_i a_{ij}, j \in J_1$$

4. Calculate weighted decision matrix

$$\tilde{V} = (v_{ij}), i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

Where

$$\tilde{v}_{ij} = \tilde{v}_{ij} \otimes \tilde{w}_j, i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

5. Determine positive and negative ideal solutions

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \tilde{v}_3^+, \dots, \tilde{v}_n^+)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \tilde{v}_3^-, \dots, \tilde{v}_n^-)$$

Where

$$\tilde{v}_1^+ = (1, 1, 1, 1)$$

$$\tilde{v}_1^- = (0, 0, 0, 0)$$

6. Calculate distances between actual decisions and positive and negative ideal solutions

$$d_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}^+, \tilde{v}_j^+) \quad j=1,2,\dots,m$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}^-, \tilde{v}_j^-) \quad j=1,2,\dots,m$$

Where distance is calculated by formula

$$D(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{4} [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2]}$$

7. Calculate closeness coefficient for all alternatives

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+}, i = 1, 2, \dots, m$$

8. Determine acceptance level of decisions.

Table 3. Acceptance Criteria

Closeness Coefficient (CC _i)	Evaluation
CC _i ∈ [0,0.2)	Not recommended
CC _i ∈ [0.2,0.4)	Recommended with high risk
CC _i ∈ [0.4,0.6)	Recommended with low risk
CC _i ∈ [0.6,0.8)	Acceptable
CC _i ∈ [0.8,1.0)	Accepted and preferred

9. Select decision with maximum of closeness coefficient.

Practical example.

As practice problem we consider equipment selection problem with following 4 criteria and 3 alternatives:

- C1- price
- C2- noise level
- C3- usability
- C4- dimension

As seen for C3 optimal decision is maximum for other three criteria is minimum.

Consider application of fuzzy TOPSIS method for this problem. All computations were performed in Ms Excel.

1. Presentation of decisions in linguistic decision matrix

	C ₁	C ₂	C ₃	C ₄
A ₁	VG	G	VG	MP
A ₂	MP	G	G	VG
A ₃	G	VG	MP	G

The vector of criteria importance is presented as follows

$$w = (ML, H, VH, H)$$

2. Convert linguistic presentation in trapezoidal fuzzy numbers

	C ₁	C ₂	C ₃	C ₄
A ₁	(9, 9.25, 9.5, 10)	(7, 8, 9, 9.25)	(9, 9.25, 9.5, 10)	(4, 5, 7, 8)
A ₂	(0.4, 0.5, 0.7, 0.8)	(7, 8, 9, 9.25)	(4, 5, 7, 8)	(9, 9.25, 9.5, 10)
A ₃	(7, 8, 9, 9.25)	(9, 9.25, 9.5, 10)	(4, 5, 7, 8)	(7, 8, 9, 9.25)

$$w = \begin{matrix} (0.4, 0.5, 0.7, 0.8) & (0.5, 0.6, 0.75, 0.85) & (0.6, 0.7, 0.8, 0.9) & (0.5, 0.6, 0.75, 0.85) \end{matrix}$$

3. Calculate normalized fuzzy decision matrix by corresponding formulas

	C_1	C_2	C_3	C_4
A_1	(0.40,0.42,0.43,0.44)	(0.76, 0.78,0.88,1)	(0.9,0.93,0.95,1)	(0.5,0.57,0.8,1)
A_2	(0.5, 0.57, 0.8, 1)	(0.76,0.78,0.88, 1)	(0.7,0.8,0.9,0.93)	(0.4,0.42,0.43,0.44)
A_3	(0.43,0.44,0.5,0.57)	(0.7,0.74,0.76, 0.78)	(0.4,0.5,0.7,0.8)	(0.43,0.44,0.5,0.57)

4. Calculate weighted normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4
A_1	(0.16,0.21,0.3, 0.36)	(0.38,0.47,0.66,0.8)	(0.54,0.65,0.76,0.9)	(0.25,0.34,0.60,0.85)
A_2	(0.2,0.29,0.56,0.8)	(0.38,0.47,0.66,0.8)	(0.42,0.56,0.72,0.83)	(0.2,0.25,0.32,0.38)
A_3	(0.17,0.22,0.35,0.46)	(0.35,0.44,0.57,0.62)	(0.24,0.35,0.56,0.72)	(0.22,0.27,0.38,0.49)

5. Calculate distance between decisions and positive and negative ideal solutions

	C_1	C_2	C_3	C_4
$d(A_1, A^+)$	1.3	0.73	0.49	0.87
$d(A_2, A^+)$	0.95	0.73	0.62	1.25
$d(A_3, A^+)$	1.22	0.86	0.92	1.16
$d(A_1, A^-)$	0.52	1.15	1.37	1.1
$d(A_2, A^-)$	1.02	1.15	1.25	0.57
$d(A_3, A^-)$	0.62	0.97	0.98	0.68

After calculating the distances between the alternatives and the fuzzy positive and fuzzy negative ideal solutions, we calculate the closeness coefficients for the all alternatives. The results is presented below

	d_i^+	d_i^-	CC_i	Ranking
A_1	3.40	4.14	0.55	1
A_2	3.55	3.99	0.53	2
A_3	4.16	3.25	0.44	3

According at the acceptance criteria of alternatives, all alternatives are determined as "Recommended with low risk". Since the closeness coefficients are ranked from the biggest to the smallest, as $CC_1 > CC_2 > CC_3$, so alternative A_1 is optimal.

Conclusions. The article is devoted to the problem of multi-criteria decision making for software selection. The analysis of existing methods for solving this problem is given. The fuzzy TOPSIS is used as a method for solving this problem. The issues of practical implementation of this method are discussed in details.

As practical problem the equipment selection problem with 4 criteria and 3 alternatives is considered. The results of the solution at all stages are presented.

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