COMPUTER SCIENCE

MATHEMATICAL MODEL FOR MANAGEMENT OF THE MUNICIPAL AUTHORITIES

A. I. Mehdialiyev, p.h.d., **H. M. Misirov,** *master student*
Academy of Public Administration under the President of the Republic of Azerbaijan
Baku, Azerbaijan

ABSTRACT

In the presented paper to solve the problem was used the "Analytic Hierarchy Process" method developed by Tomas L. Saaty. AHP is one of the most popular analytical techniques for complex decision-making problems. In this method a decision-making problem decomposes into a system of hierarchies of objectives, attributes (or criteria), and alternatives. Then to obtain optimum solution uses judgements of experts with a special scale for measuring non-quantitative and method of establishing priorities.

Problem statement. Despite the fact that in the history of Azerbaijan had many examples of self-government, municipalities as elective form of self-government can be taken as a new Institute for us. This form of self-government was established by law in 1995, in the country's Constitution. But the life of this form of self-government was only possible in 1999, after the first election in the municipalities. Thirteen-year-old experience in this area has shown that there are some problems in the organization and functioning of such a form of local government. There are problems with the executive authorities in respect of the formation of municipal property. Low participation of the local population in the municipality. There are big problems in technical equipment of municipalities; the activities of these bodies are not computerized. In our view, to identify the causes of the barriers to the establishment and effective functioning of municipalities to this problem must be approached systematically. To this end, to undertake a systematic analysis of the situation and work out effective measures for improvement and further development of the municipalities of the Republic.

In recent times in the scientific literature, most often on the Internet there are various applications of the "Analytic Hierarchy Process" (AHP) "method American mathematician Thomas L. Saaty [1]. Interestingly, among the various applications of this method, you can deal with a wide variety of tasks. T. Saaty itself applied this method to process different tasks in different countries: in the planning of the transport system in the Sudan, in the Mexican brewing industry, in the field of nuclear energy (Canada), in the field of aircraft industry (Israel), to predict the development of higher education in the United States, etc. This report addresses the challenge of improving municipal management in Azerbaijan and its prediction for the future. The diversity of tasks shows the versatility of the method of analysis of hierarchies. Therefore, it is no coincidence that this method was applied in making such tasks as building a

scenario for the development of science in high school [3], support of main oil pipeline Baku-Tbilisi-Ceyhan pipeline [4], and ensuring transparency in the distribution of oil revenues in Azerbaijan [5]. By the author of this report also had been attempted to develop mathematical model for definition of mentality of a given nation on the basis of its proverbs and sayings with AHP [6,7].

In this report considered the questions of improving and forecasting of municipal management. Know that to build any mathematical model, first you need to have a broad knowledge base. In our view, to build an optimal model of municipal management, primarily to study the structure of municipalities throughout the country, summarize the experience of advanced municipalities. To this end should collect all information on municipal authorities and created the database. To build a mathematical model of another source of information can be the outcome of municipal workers and population of sociological research. In this direction, in the country is carried out certain work. For example, one can mention the work [8][1].

The mathematical apparatus to solution of task.

In the method AHP, in the first stage requires to create a hierarchical structure for the problem under consideration. Assume that the expert group created the following hierarchical structure for our task (Figure 1):
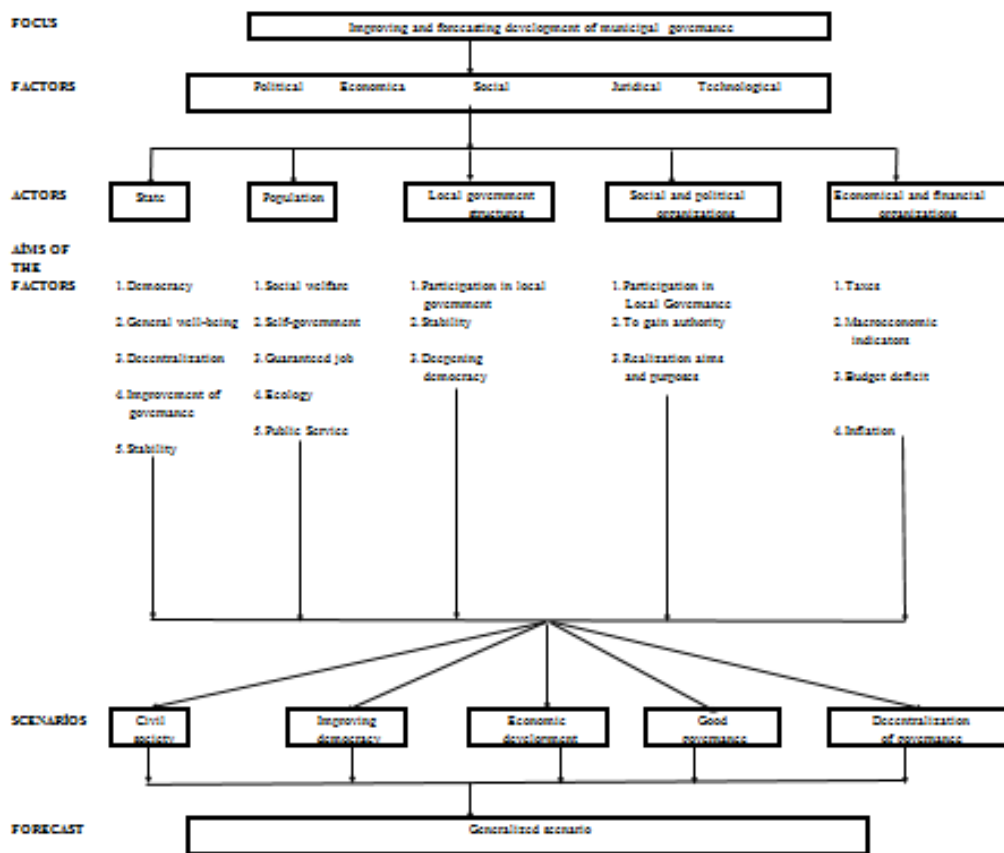

Fig.1. Hierarchical structure of the problem

Form, based on statistical reports and other quantitative information about municipal activities are only part of the information management tasks for municipalities. The proposed us a mathematical model as the data also requires the collection and other qualitative information. A processing of information by the expert group is carried out on the basis of a specially designed scale for this purpose Saaty's nine-point scale for relative importance [1] (Table 1).

A mathematical model is constructed for this task and appropriate software is created. The process of processing expert judgment and constructing a generalized scenario is fully automated. The expert evaluation is processed in two stages. For this purpose, input forms are developed for each level of the structure based on a hierarchical structure. At the first stage, input forms filled with experts for

the first, second, third levels of the hierarchical structure are formed in the form of matrices, and the eigenvectors of these matrices are found. The matrix formed from the eigenvectors of the second level on the right is multiplied by the eigenvector obtained for the first level. To this end, the weights of the selected factors are multiplied by their own respective goals. Of these values, the most significant are taken and they are normalized. It is these normalized values that are considered to be the most important goals for the problem considered. These values are reported to the experts and ask them to fill in the new input forms for each proposed scenario. All scenarios are compared with each other for each important goal in a manner similar to the rule described above. Then the matrix formed from the eigenvectors of the most important goals of the actors is multiplied by the normalized values of the weight coefficients. Thus, the second stage of expert evaluation processing is completed, as a result of which we determine the weighting coefficients of the scenarios.

Table 1. Saaty's nine-point scale for relative importance

Stage of Scale	Verbal Judgement	Characteristics
1	Equal Importance	Two activities contribute equally
3	Moderate Importance	Experience and judgement moderately favour one activity over another
5	Strong Importance	Experience and judgement strongly favour one activity over another
7	Very Strong Importance	An activity is strongly favoured and its dominance demonstrated in practice
9	Absolute Importance	The evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values bet-ween the two dja-cent judgements	When compromise is needed

To this end, the following were identified as the main factors influencing the problem: political, economical, social, juridical, technological, next, the main actors of the problem: state, population, local government structures, social and political organizations, economical and financial organizations. Identified and clearly set out the main objectives of each actor in this issue. As an example, here we list the main objectives of the state:

1. Democracy
2. General well-being
3. Decentralization
4. Improvement of governance
5. Stability

To build a generic script in this task, the following alternative scenarios were selected: Civil society, Improving democracy, Economic development, Good governance, Decentralized governance.

Let's say that as a result of the expert estimation on the basis of the scale of relative importance are numbers: $\omega_1, \omega_2, \omega_3, \ldots \omega_n$. We form from these numbers the following matrix:

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \ldots & a_{1,n} \\ 1/a_{12} & 1 & a_{23} & \ldots & a_{2,n} \\ 1/a_{13} & 1/a_{23} & 1 & \ldots & a_{3,n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1/a_{1,n} & 1/a_{2,n} & 1/a_{3,n} & \ldots & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \varpi_1/\varpi_1 & \varpi_1/\varpi_2 & \varpi_1/\varpi_3 & \ldots & \varpi_1/\varpi_n \\ \varpi_2/\varpi_1 & \varpi_2/\varpi_2 & \varpi_2/\varpi_3 & \ldots & \varpi_2/\varpi_n \\ \varpi_3/\varpi_1 & \varpi_3/\varpi_2 & \varpi_3/\varpi_3 & \ldots & \varpi_3/\varpi_n \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \varpi_n/\varpi_1 & \varpi_n/\varpi_2 & \varpi_n/\varpi_3 & \ldots & \varpi_n/\varpi_n \end{bmatrix}$$

If to designate elements of this matrix by a_{ij}, $i, j = 1, 2, \ldots n$. Then we will obtain:

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \ldots & a_{1,n} \\ 1/a_{12} & 1 & a_{23} & \ldots & a_{2,n} \\ 1/a_{13} & 1/a_{23} & 1 & \ldots & a_{3,n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1/a_{1,n} & 1/a_{2,n} & 1/a_{3,n} & \ldots & 1 \end{bmatrix}$$

Clearly, that the matrix A satisfies a reciprocal property:

$$a_{ij} = \frac{1}{a_{ji}}, \ i, j = 1, 2, \ldots, n \tag{1}$$

Let us consider an eigenvalue problem for the matrix A :

$$Ax = \lambda x \tag{2}$$

Where, λ is a eigenvalue, and $x = (x_1, x_2, x_3, \ldots x_n)$ is a eigenvector.

Is known, that for a reciprocal matrix takes place

$$\lambda_{max} \geq n \tag{3}$$

Where, λ_{max} - greatest eigenvalue, n - order of the matrix A. The equality sign takes place only for coherence matrices [1].

As noted above, for elements of each level of the hierarchical structure, the coefficients of relative importance are found as a solution of the eigenvalues and eigenvectors of the matrices of pairwise congruences. In the general case, there are strict mathematical methods for solving this problem. But, T. Saati in his book [2] proposed simple formulas for calculating eigenvalues and eigenvectors. These formulas are proposed in Figure 2.

$$X_1 = (1 * (W_1/W_2) * \ldots * (W_1/W_n))^{1/n}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$X_n = ((W_n/W_1) * \ldots * (W_n/W_{n-1}) * 1)^{1/n}$$

$$\lambda(A_1) = X_1 / \sum X_i$$

$$\lambda(A_n) = X_n / \sum X_i$$

Fig.2. Simple formulas for calculating eigenvalues and eigenvectors

Further, for each level, the consistency index (CI), determined by the formula:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

After computing the CI from values scale 1/9, 1/8, 1/7, ...1, 2, 3, ..., 9 randomly formed the coherence matrices and for different orders are calculated random index (RI). Middle RI matrices for matrices of order from 1 to 10, on the basis of 100 random samples are presented in the form of the following standard table.

Table 2. Average random number index for each size of the matrix

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0,58	0,9	1,12	1,24	1,32	1,41	1,45	1,49

In this table, the first line specifies the matrix size -n and on the second line the average RI. Dividing, CI to RI receive ration consistency (RC).

$$RC = \frac{CI}{RI}.$$

It is generally considered that for harmonised data RC must not exceed 0,1 (10%), in some cases, 0,2 (20%).

As noted above, expert evaluation by the AHP method is carried out in two stages. At the first stage, the goals most relevant to the overall goal of the system are determined and the most important ones are selected from them. Omitting the details of the calculation, we give the normalized values (Fig.3):
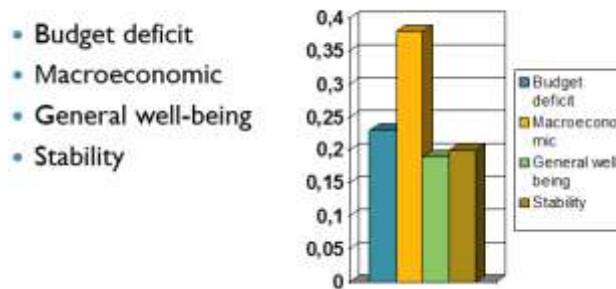

Fig. 3. Identifying the most important goals of actors

At the second stage, the alternative scenarios defined in the hierarchical structure (scheme) are compared in pairwise relative to each important goal on the basis of a scale of scale of difference for comparsion (Fig.4.):
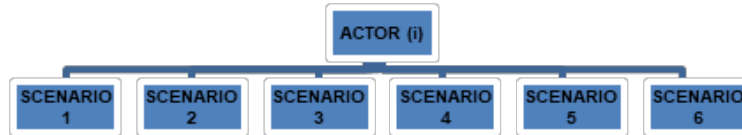

Fig. 4. Pairwise comparison of scenarios with each other

Then, according to the scheme described above, the eigenvectors of the matrices filled by experts on a pairwise comparison with respect to the selected goals are determined: budget deficit, economic indicators, common welfare, and stability. Further, multiplying the matrix formed from the vectors thus obtained by the weight vector of the most important factors. As a result, we obtain the weights of alternative scenarios.

The next step in solving the problem by the AHP method is expert estimation of the attribute and characteristics of local government based on the difference scale for comparison (Table 3):

Table 3. Scale of difference for comparison

Difference in values	Definition
0	Value does not change
2 (-2)	A small increase (decrease) in value
4 (-4)	A large increase (decrease) in value
6 (-6)	A significant increase (decrease) in value
8 (-8)	The maximum increase (decrease) in value
1,3,5,7,-1,-3,-5,-7	Intermediate values between the two judgments

Finally, based on expert assessments for attributes and characteristics of local government and calculated weighting coefficients, a generalized scale is constructed, according to the values of which the forecast of the development of the municipality bodies is built.

Conclusions. It is known that in the method of analysis of hierarchies just as when constructing the hierarchical structure of the problem, as on the pairwise comparisons of elements at various levels of a hierarchical structure of tasks are use an expert estimation. The method also allows you to define Pareto-optimality of the system. Note that when using MAH iteratively deeper reveals the essence of the problem. Repeat the process with a view to clarifying the opinions collected additional knowledge that allow you to experiment. In addition there can be somewhat inconsistent level of expert opinions. If the experts were not able to reach consensus in dealing with the problem, as the peer review can take geometric mean of alternative estimates.

In this task the input forms are filled on the basis of the scale of relative importance in two stages. The information gathered in the first phase is processed on the computer and identifies important goals. For these goals are formed new input forms and they are distributed to the experts for filling again. The information gathered in the second phase is processed on the computer also. Based on the obtained results and other information, alternative scenarios are compared on the described technique. Based on the constructed mathematical model is generalized scenario, which makes it possible to optimize the structure of municipal bodies and predict their development.

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