HYDRODYNAMIC METHODS FOR THE INVESTIGATION OF WELLS AND RESERVOIR

Abasova Inara,
Azerbaijan State Oil and Industry University, department "Computer Engineering" assistant, Azerbaijan, Baku

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ABSTRACT
The analysis of literary sources in this study showed that in order to solve the problem of the pressure recovery curve during hydrodynamic probe of the reservoir in the case of its constant porosity a number of studies were dedicated to the nonstationary solution of the deterministic Darcy equation for an unlimited reservoir or on methods of empirical or semi-empirical modeling for static errors, which is associated with a nontrivial approximation of the asymptotic part of the pressure recovery curve.
In this study, a new method for calculating the hydrodynamic parameters of an oil reservoir is proposed, taking into account all the information from the pressure recovery curve. In contrast to the existing methods, the proposed model of pressure versus time adequately describes the entire pressure recovery curve when the well is closed, which makes it possible to fully evaluate the hydrodynamic parameters of the reservoir.

KEYWORDS
PRC,
well,
viscosity of reservoir fluid,
oil reservoir,
reservoir porosity,
bottom-hole pressure.

Introduction. The analysis of the properties and characteristics of an oil reservoir is based on an estimation of the pressure recovery curve (PRC) during the shutdown of wells operating for a sufficiently long time with a constant flow rate. Using the properties of HPC and the nature of the change curve in bottom-hole pressure versus logarithmic time, $P = f(\ln t)$, its asymptotic and putting it as a linear function of time are estimated according to the basic characteristics models of the oil reservoir (coefficients of hydro-conductivity, piezo-conductivity, permeability, etc.) by model. d.). Two methods are used in order to determine the coefficient of productivity ($k_P$) or reservoir pressure in the unstable conditions.

The first method is based on the determination of reservoir pressure or pressure in the supply circuit ($P_k$) by PRC in the form of the dependence $P = f(t)$ [1].

The second method is based on the dependence of the downhole pressure of a stopped well on the logarithmic time $P_0 = f(\ln t)$.

Using the first method, the author of work [1] has developed mathematical models for describing experimental data on PRC deterministic time points, where the initial part of the curve is described by a third-degree polynomial and the final part (the asymptotic part of the curve) is approximated by a linear logarithmic dependence by using the tangent method. However, in this method [1] there is uncertainty in determining the length of sections and the transition point from one section to another. In addition, such an estimation of $P_k$ leads to errors connected with a nontrivial approximation of the asymptotic part of the PRC, since asymptotic is not a logarithmic non-linear
function, but a non-linear function of exponential nature. Therefore, a third degree polynomial is an incomplete decomposition of the true, by nature, exponential dependence in a Taylor series. Moreover, in this work, the well shutdown time for the removal of experimental pressure curves data according to PRC is 15.68 days, which is an ideal case where $P_{c0}$ change is almost restored to $P_k$. In this regard, in the paper the following mathematical model is proposed for the processing of proper data obtained from the study of wells listed in Table 1:

$$P_{ct} = P_{c0} + \Delta P \left[ 1 - \exp \left( - \frac{t}{T} - 0.1556 \left( \frac{t}{T} \right)^2 + 0.07 \left( \frac{t}{T} \right)^{1/2} \right) \right],$$  \hspace{1cm} (1)$$

where $P_{c0}$, $P_{ct}$ is the bottom-hole pressure before the well stops and its current value, respectively; $t$ is the current time, $h$; $T$ is the characteristic time of the PRC, determined as a result of the tangent to the experimental curve (Fig. 1), which is specified in the identification process of the model (1).

Table 1. Experimental data on PRC at fluid flow rate $Q = 5m^3/s$

<table>
<thead>
<tr>
<th>Data</th>
<th>Measurement time, h:</th>
<th>Measured pressure in the middle of the perforation interval, atm.</th>
<th>Dynamic level, m</th>
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<td>11:03</td>
<td>89.8</td>
<td>2079</td>
</tr>
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<td>08.03.11</td>
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</tr>
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</tr>
<tr>
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<td>117.1</td>
<td>1800</td>
</tr>
<tr>
<td>11.03.11</td>
<td>15:38</td>
<td>118.0</td>
<td>1790</td>
</tr>
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</tr>
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<td>1770</td>
</tr>
<tr>
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</tr>
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<td>123.5</td>
<td>1734</td>
</tr>
<tr>
<td>14.03.11</td>
<td>16:00</td>
<td>123.9</td>
<td>1730</td>
</tr>
<tr>
<td>15.03.11</td>
<td>8:50</td>
<td>124.1</td>
<td>1728</td>
</tr>
<tr>
<td>16.03.11</td>
<td>8:51</td>
<td>124.5</td>
<td>1724</td>
</tr>
</tbody>
</table>

As it is seen from the figure 1, $P_{c0} - P_{c0} = 3.0MPa$ ($P_{c0}$ is bottom-hole pressure at $t = \infty$ or reservoir pressure).

Therefore, taking into account these values of formula [1], the following form is obtained:

$$P_{ct} = P_{c0} + 3.0\left[ 1 - \exp \left( - \frac{t}{T} - 0.0333t - 5.1087 \cdot 10^{-3} r^2 + 2.3333 \cdot 10^{-3} r^{1.5} \right) \right].$$  \hspace{1cm} (2)$$

As follows from the proposed formula [2], the value is $t = \infty$ $P_{ct} = P_k$.

As it is seen from Fig. 1, the approximation of experimental data according to formula [2] has high accuracy with a relative error of 5.1%

$$tg\alpha = i = \frac{\Delta P_k}{tg t_2 - tg t_1}.$$  

As mentioned above, the second method of determining reservoir pressure by PRC is to use the dependency $P_{ct}$ from ln $t$, which allows to determine not only the values of $P$ (or the coefficient of reservoir productivity), but also other characteristics of the oil reservoir.
Fig. 1. Comparison of experimental data on PRC (points taken from Table 1) with calculated values (solid curve).

At the same time, using the properties of PRC and the filtration equation of a compressible fluid in an elastic reservoir (equation of transmissibility) in cylindrical coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P}{\partial r} \right) = \frac{1}{\chi} \frac{\partial P}{\partial t}$$

(3)

the following formula is obtained:

$$\Delta P = \frac{10^3 Q \mu}{\pi kh} E \left( - \frac{r_r^2}{4 \chi t} \right)$$

where \( r \) is the current radius of the reservoir contour; \( P \) is the pressure of the supply circuit at distances \( r \) from the well; \( \Delta P = P_e - P_{e0} \) is the coefficient of piezo-conductivity of the reservoir in the well area (averaged), m²/c; \( E \left( - \frac{r_r^2}{4 \chi t} \right) \) is an integral exponential function; \( r_r \) is the reduced well radius.

There is also a solution of equation [3] in the form of the Macket formula (2-5):

$$\Delta P = \frac{Q \mu b}{4 \pi kh} \ln \left( - \frac{2.25 \chi t}{r_r^2} \right)$$

(4)

where \( b \) is the volume coefficient; \( k \) is permeability, m²; \( h \) is the thickness of the reservoir, m; \( \mu \) is dynamic viscosity of the reservoir fluid, Pa·c; \( Q \) is well flow rate before stopping with a constant flow rate, m³/c; \( \Delta P \) is the increase in the bottom-hole pressure of a stopped well, Pa.

This formula is obtained for a uniform unlimited reservoir. After some transformations, the Macket formula (4) is represented as a straight line, the so-called MDH formula (Miller-Deys-Hutchinson), constructed in semi-logarithmic coordinates \( \Delta P \sim \ln t \) [2-6],

$$\Delta P = \frac{Q \mu b}{4 \pi kh} \ln \left( \frac{2.25 \chi t}{r_r^2} \right) + \frac{Q \mu b}{4 \pi kh} \ln t = A + \ln t$$

(5)

here \( A = \frac{Q \mu b}{4 \pi kh} \ln \frac{2.25 \chi t}{r_r^2} \) is a section, clipped on the Y-axis, Pa;

\( i = \frac{Q \mu b}{4 \pi kh} \) is angular coefficient.

The geometric interpretation of formula [5] in decimal logarithms is shown in Figure 2 [5].
The disadvantages of the outlined methods are, firstly, that they describe only a linear section in semi-logarithmic coordinates or only a certain part of the entire curve [5,6] is straightened, secondly, in most cases, especially for wells with less yield (with a limited time to stop the well 8 days), there is no rectilinear section on the end part of the PRC and the restoration of bottom-hole pressure to reservoir [5]. In addition, as mentioned in [6], when applying existing methods, the very inverse problem of determining the filtration characteristics of a reservoir according to PRC turns out to be incorrectly set: its solutions are unstable with respect to errors that are inevitably contained in the measurements. In particular, the instability manifests itself in conditions of small samples, when it is possible to straighten only a small portion of the PRC in a semi-logarithmic coordinates. The authors of work [6] note that “very often it is also necessary to process the so-called unrestored PRC obtained in experiments that were interrupted for technical reasons or because of the desire to reduce oil losses due to well shutdown.” This is confirmed by the experimental data presented in [6] (Fig. 3) and the data from our own research, shown in Fig. 1. Straight line 1 in Fig. 3 was conducted by the least squares method through a straightened section AB. Straight lines 2 and 3 are bounded by the area in which the straight lines are obtained from model samples. The position of straight line 1 shown in Fig. 3, as noted by the authors [6], is only a random result obtained with a given randomly implemented sampling of data. If another PRC had been removed, the experimental points could have been located differently due to errors, and this would lead to a completely different straight line. Moreover, small errors in determining $\Delta P$ lead to significant errors in determining the reservoir filtration characteristics. According to [6], the variation in determining the hydraulic conductivity of the reservoir is 200%.

Fig. 2. Dependency of curve $\Delta P$ on $\lg t$: the equation of a straight line corresponds to formula (5)

Fig. 3. Experimental data on PRC taken from work (6)
The purpose of the proposed study is to develop a method for determining the filtration characteristics of an oil reservoir based on an adequate mathematical description of experimental data for PRC shown in Fig. 1 and 3.

\[
P_{ct} = P_k - (P_c - P_{ct0}) \exp\left[-m(\ln t)^{n-1}\right]
\]  

and on the base of this formula, the determination of reservoir characteristics (\(m\) and \(n\) are exponential coefficients). The calculated values of the curves shown in Fig. 4 and 5 are based on the indicators of the proposed mathematical model [6] for an adequate description of the experimental data given in Table. 2.

![Graph 4](image1)

**Fig. 4. Comparison of the experimental pressure recovery curve (6) with the calculated values (6)**

\(n = 2, m = 0.05\)

**Table 2. The values of the coefficients included in equation (6)**

<table>
<thead>
<tr>
<th>The number of images</th>
<th>Indicators</th>
</tr>
</thead>
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<td>(P_k), MPa</td>
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<tr>
<td>4</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>12.5</td>
</tr>
</tbody>
</table>

The advantage of formula (4) in comparison with the existing formulae is that it describes the entire PRC by a single equation. This allows to estimate the hydrodynamic properties of the oil reservoir without additional tangential and approximations of the quasilinear part of the curve. As follows from fig. 4 and 5, the proposed mathematical model has an improved structure and adequately describes the experimental data.

![Graph 5](image2)

**Fig. 5. Comparison of the experimental pressure recovery curve (Table 1. Fig. 1) with the calculated values (6)**

\(n = 2, m = 0.02\)

In contrast to the existing methods, including the Horner method (3), based on the methods of tangent or three points, which use only the linear logarithmic section (if it exists) of the semi-logarithmic PRC to
determine the filtration characteristics of the reservoir, the average values of the reservoir hydrodynamic characteristics are estimated more precisely and reliably in the proposed method of the following algorithm.

1. The derivative of $P_{ct}$ from $\ln t$ is determined in the following form:

$$\frac{dP_{ct}}{d\ln t} = m(n+1) - (P_c - P_{ct})(\ln t)^\gamma \exp\left[-m(\ln t)^{\gamma+1}\right]$$

2. By means of equation:

$$\frac{dP_{ct}}{d\ln t} = \frac{Qub}{4\pi kh} = \frac{Qb}{4\pi \xi}.$$

hydraulic-conductivity of $\xi$ oil reservoir is determined:

$$\xi = \frac{Qb}{4m(n+1)(\ln t)^\gamma \exp\left[-m(\ln t)^{\gamma+1}\right]}.$$ 

3. For specific data, the mean $E$ is calculated according to the formula:

$$\bar{E} = \frac{\sum_{i=1}^{N} \xi_i}{N},$$

where $N$ is the total number of the calculated data.

For the experimental data obtained by the authors of this study in Figure 5, where $Q = 5.79 \cdot 10^{-5} m^3/s$, $h = 12 m$, $\mu = 3 \cdot 10^{-3} Pa/s$ the followings are obtained:

$$\xi = \frac{\sum_{i=1}^{12} \xi_i}{12} = 4,941 \cdot 10^{-11} \frac{m^3}{Pa \cdot s} = 4,941 \frac{D \cdot sm}{sP} \cdot \delta$$

$D = 10^{-12} m^3$.

4. The average value of permeability is determined in the following form:

$$k = \frac{\xi \mu}{h} = \frac{4,941 \cdot 10^{-11} \cdot 3 \cdot 10^{-3}}{12} = 12,3675 mD.$$ 

5. The average value of piezo-conductivity coefficient is determined as follows:

$$\chi = \frac{k}{\mu(m_{ct} + \beta_f)} = \frac{12,3675}{3 \cdot 10^{-3}(0,18 \cdot 6 \cdot 10^{-10} + 10^{-10})} = 1,98 \cdot 10^{-2} m^2/s = 198 sm^2/s.$$ 

where $\beta_f$, $\beta_i$ are the coefficients of the elastic capacity of the fluid and reservoir; $m_{ct}$ is the reservoir porosity.

**Conclusions.** Analysis of existing methods showed that, using the properties of the PRC or, mainly, its asymptotic and adding it a linear function of time, the basic average statistical properties of the oil reservoir (coefficient of piezo-conductivity, permeability, hydro-conductivity) are usually estimated using the model. It should be noted that such an estimation of the basic properties of the oil reservoir leads to large statistical errors associated with a non-trivial approximation of the asymptotic part of the restored pressure curve, since the asymptotic is not a linear function, but a weak non-linear function of exponential nature.

On this basis, this study proposed a method for determining the filtration parameters of an oil reservoir in different ways allowing to take into account the entire pressure recovery curve, for which a model was proposed that fully describes the dependence of pressure on time with a closed well.

**REFERENCES**

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