

COMPUTER SCIENCE

NUMERICAL AND ANALYTICAL DIAGRAM OF A DISTRIBUTED SIMULATION OF DYNAMIC SYSTEMS

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ABSTRACT

The work is dedicated to the construction of numerical-analytical method of designing efficient algorithms for the solution of problems in economics and engineering. Using a priori information about the smoothness of the solution, great attention is paid to the construction of high-accuracy solutions. The proposed approach eliminates recurrent structure calculations unknown vectors decisions, which leads to the accumulation of rounding errors. Parallel form of the algorithm is the maximum, and therefore has the shortest possible time the implementation on parallel computing systems. Most conventional algorithms for solving these problems (sweep techniques, decomposition of the matrix into a product of two diagonal matrices, doubling, etc.) when multiple processors work typically no faster than if a single processor. The reason for this is substantial sequence computations of these algorithms.

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Introduction. Problems that arise in front of the economists, are complex in their majority. This is because they depend on many factors, which not only influence each other, but also determined time dependences [1]. For this reason, this class of problems studied by means of economic and mathematical modeling [2, 3]. The mathematical model allows to take into account a variety of parameters that affect the economic system as a whole.

However, the economic and mathematical modeling is often a situation arises when the system under study has an extremely complex structure. Quite often this is due to the multidimensionality of their description. Multivariate models are used for marketing research and management. In addition to such problems include the problem of segmentation and market forecasting, study of the economic depression, the analysis and forecasting of social and economic phenomena, and others. We have a class of problems is extremely important for the economy as a whole, in this regard, the development of effective methods of their solution seems relevant scientific and practical problem.

In recent decades, studies of the dynamics of economic systems, there is a steady trend of transition to continuous time [4, 5]. The economic problems of the class of continuous-time to adapt

the methods and models, experience study of linear and nonlinear dynamical systems, gained in technical sciences (and especially in the theory of automatic regulation) [6].

At the same time, an important feature of the solution given class of problems is their high computational complexity. Therefore, their effective research tasks can be carried out only on the basis of multiprocessor systems [7, 8].

On the other hand, through the use of finite difference schemes for a significant acceleration of computing these problems is achieved by the effect of parallelization. Here the individual noteworthy numerically-analytical algorithms for solving applied problems. Higher speedup compared with the finite-difference approach can be achieved using analytical solutions that allow you to perform calculations simultaneously and in parallel on all temporary layers and do not use at the same time combined memory [9]. However, the most promising approach to mathematical modeling of applied problems of economics and technology should be considered the use of numerical and analytical solutions [10, 11]. Thus, distributed application modeling problems of the economy based on high order schemes is an important and urgent task.

Analysis of recent research and publications. Currently, macroeconomic processes studied as transient processes in dynamic systems [6]. Therefore, the study of macroeconomic processes carried out by means of mathematical methods and models, primarily with the help of the theory of dynamical systems (mainly automatic control theory), which is based on the apparatus of differential equations and Laplace transforms. In the study of transients in unstructured macroeconomics uses a dynamic model of Keynes and Samuelson-Hicks model [3]. Moreover, the processes of accumulation of money and material modeling of the dynamics of securities is treated as the cost of large systems [5].

Note that this class of problems, as a rule, is described by differential equations and can be solved through the application of difference schemes apparatus, the essence of which is that carried out the replacement of derivatives by difference relations. Thus from the point of view of numerical algorithms for solving differential equations is distributed on explicit and implicit schemes [12]. The explicit scheme the value of the unknown function evaluated successively, layer by layer. In this context, for parallel computing, this approach can not be used. Implicit schemes allow to conduct calculations with a big step without a significant loss in accuracy, but this approach requires more computation. The above analysis shows that the methods of solving problems of analysis capabilities. At present, there have been certain trends in the development of numerical and analytical methods with complex logical structure, but they are compared with piecewise-difference methods of higher order accuracy and the possibility of constructing algorithms with adaptation for orders of approximation [10, 11]. From the point of view of calculating this approach is a cumbersome, but it shows a kind of benchmark for comparison with other practices. However, despite the fact that the computer experiment is carried out on a multiprocessor system, it can be argued that the fact hindered the development of numerical and analytical approach, is now losing its relevance.

It should be noted that today the solution of complex, large-volume tasks requires powerful computers and is characterized by a parallel word, that is, there are parallel computers, computer systems, parallel computing techniques, etc. [13-16]. The appearance in the computer systems of new means of communication, more advanced element base stimulated the development of HPC based on standard technologies and public components [7, 8].

In this paper, for the computational experiments are used so-called "blade" server solutions for multiprocessor systems [8]. On the basis of IB network technologies have been implemented "blade" server solution multiprocessor system are installed in one housing in which several similar parent modules. Practice shows that blade systems are more compact and easy to maintain, and their implementation is not much more expensive compared to the multi-processor computer systems. The main features of its design architecture presented in [7].

Thus, it can be argued that by now disappeared fundamental problems in a potentially infinite increase in peak performance computers. But the really serious problem is and how to use this enormous potential. In this paper, the possibility of constructing the maximum parallel computing algorithms in problems of technology and economics.

Unsolved part of the problem. Existing methods for solving the problems of the economy and technology are not always suitable for reasons of accuracy, speed, memory requirements, the structure of algorithms, applicability for multi-processor computer systems. In this context, there are new ideas and implemented in the field of computational mathematics. Ultimately, for more sophisticated mathematical models needed to design new methods of implementation of numerical experiments.

The purpose of the study. The purpose of this work is to construct the most parallel algorithms for solving the problems of the economy and technology, which are described by dynamic models. In this case we deal with the problems of mathematical modeling of this class of problems on parallel computing systems of cluster type. Most conventional algorithms for solving these problems (sweep techniques, decomposition of the matrix into a product of two diagonal matrices, doubling, etc.) when multiple processors work typically no faster than if a single processor. The reason for this is substantial sequence computations of these algorithms.

Basic results of research. Creation of parallel computing systems required the development of mathematical concepts for constructing parallel algorithms, i.e. algorithms adapted for implementation in these systems. As the basis for constructing the parallel algorithm we can take both: a sequential algorithm and the task itself as well [9, 14]. The most sensible at parallelization of sequential algorithm is pragmatic approach; actually sequential algorithms detect common elements which further are transformed to a parallel form.

The numerical and analytical schemes consider the example of the boundary value problem for the heat equation with constant coefficients. E.g. we want to find a solution in the area $\{0 \leq x < x_L, 0 < t \leq T\}$:

$$\frac{\partial Y}{\partial t} = \frac{\partial^2 Y}{\partial x^2}, \tag{1}$$

which satisfies the initial condition

$$Y(x, 0) = \varphi(x) \tag{2}$$

and the boundary conditions

$$Y(0, t) = \mu_0(t), \quad Y(x_L, t) = \mu_L(t) \tag{3}$$

Here $\mu_0(t)$, $\mu_L(t)$, $\varphi(x)$ – are given functions. It is known that under certain assumptions of the smoothness problem (1) - (3) has a unique solution [4].

We propose to apply the net on value x with the step between nodes equal to.

$$Dx1_p = \frac{x_L}{2m}, \quad p = \overline{1, 2m-1}, \quad m \in Z, \tag{4}$$

where m – is the integer parameter sampling. For uniformly distributed nodes

$$\begin{cases} Dx1 = x_p - x_{p-1} = const, \\ x_p = x_{p-1} + p \cdot Dx1, \quad p = \overline{1, 2m-1}. \end{cases} \tag{5}$$

On the basis of prior information required function is represented as a Taylor series:

$$Y_{p+\varepsilon_x, 1}(t, x) = \sum_{n=0}^{\infty} \varepsilon_x^n \cdot Y_{p, n+1}(t), \tag{6}$$

with

$$\begin{cases} \varepsilon_x = \frac{x - x_p}{x_{p+1} - x_p} \in [+1, -1], \\ Y_{p, n+1} = \frac{Dx1^n}{n!} \cdot \frac{\partial Y}{\partial x^n} \Big|_{x=x_p}. \end{cases}$$

After agreement (6) with equation (1) and equating the coefficients of equal powers we receive ε_x^n , the system of ordinary differential equations (SODE)

$$Y'_{p, n+1}(t) = \frac{(n+1)(n+2)}{Dx1^2} \cdot Y_{p, n+3}(t) \tag{7}$$

having the form of Cauchy

$$Y_{p,n+1}(0) = \varphi_{p,n+1}, \tag{8}$$

where $\varphi_{p,n+1}$ – are the known values of the Taylor component of the initial function (2).

Let restrict a finite number of terms $n = N$ series in the right side of the Taylor series (6), so we obtain

$$Y_{p+\varepsilon_x,1}(x,t) = \sum_{n=0}^N \varepsilon_x^n \cdot Y_{p,n+1}(t), \tag{9}$$

where N – is the integer number. To approximate equation (1) in the point (x_p, t) we will consider the closing connection

$$\begin{cases} Y_{p,N+1} \\ Y_{p,N} \end{cases} \tag{10}$$

We suppose that in (9) $\varepsilon_x = \pm 1$ and thus we obtain on the three-point template the system of two algebraic equations

$$\begin{cases} Y_{p,N+1} + Y_{p,N} = \left[Y_{p+1,1} - \sum_{n=0}^{N-2} Y_{p,n+1} \right], \\ Y_{p,N-1} - Y_{p,N} = (-1)^N \cdot \left[Y_{p-1,1} - \sum_{n=0}^{N-2} (-1)^n \cdot Y_{p,n+1} \right]. \end{cases} \tag{11}$$

We find

$$\begin{cases} Y_{p,N+1} \\ Y_{p,N} \end{cases} = \frac{1}{2} \cdot \left\{ \left[Y_{p+1,1} \pm (-1)^N \cdot Y_{p-1,1} \right] - \sum \varphi_n^\pm \cdot Y_{p,n+1} \right\}, \tag{12}$$

where

$$\varphi_n^\pm = 1 + (-1)^{n+N}, \quad N = 2, 3, 4, \dots \tag{13}$$

are normalizing factors.

For $N = 2$ $n=0, 0$ we have

$$\begin{cases} Y_{p,2} = \frac{1}{2} \cdot \left[Y_{p+1,1} - Y_{p-1,1} \right], \\ Y_{p,3} = \frac{1}{2} \cdot \left\{ \left[Y_{p+1,1} + Y_{p-1,1} \right] - 2 \cdot Y_{p,1} \right\} \end{cases} \tag{14}$$

After substituting (14) into (7) we obtain the SODE

$$Y'_{p,1}(t) = \frac{1}{Dx1^2} \cdot \left\{ \left[Y_{p+1,1}(t) + Y_{p-1,1}(t) \right] - 2Y_{p,1}(t) \right\}, \quad p = \overline{1, 2m-1}, \tag{15}$$

where $\{Y_{0,1}(t), Y_{2m,1}(t)\}$ are the boundary functions of the first kind

For $N = 3$ and the significance of the relations (7) and (14) we obtain the higher-order SODE

$$\begin{cases} Y'_{p,1}(\tau) = \frac{1}{2 \cdot Dx1^2} \cdot \left[Y_{p+1,2}(\tau) - Y_{p-1,2}(\tau) \right], \\ Y'_{p,2}(\tau) = \frac{1}{Dx1^2} \cdot \left[Y_{p+1,2}(\tau) + Y_{p-1,2}(\tau) - 2Y_{p,2}(\tau) \right] \end{cases} \tag{16}$$

where

$$\begin{cases} Y_{0,2}(\tau) = Dx1 \cdot gW(\tau), \\ Y_{2m,2}(\tau) = Dx1 \cdot gL(\tau), \end{cases} \tag{17}$$

are known boundary functions of the second kind.

Note that the developed approach includes conventional finite-difference methods in a special case. Scheme (15) coincides with the classical Dirichlet problem, and the circuit (16) with the Neumann problem. The problem (16) is characterized by the fact that the transmission of information on the boundaries of the area in the natural scheme is implemented through internal point accurately without reducing the order of approximation.

With the increase of N - order reducing the approximation orders of closing bonds (12) also increases. Note that the integration of SODE (15) - (17) having the Cauchy form with explicit methods is the most advanced procedure. The variety of standard programs allows us to consider this process as an elementary. From the point of view of cost effectiveness depending on operations' number for the mentioned above methods cannot be improved.

The developed numerical and analytical procedure for discretization can be simply generalized to other types of differential equations of mathematical physics. In particular, in the stationary problems it is easier to localize features in the regions of smoothness using schemes of high order accuracy.

The value of the order of approximation in conjunction with carrying out the calculation on the shredder grids allows to focus in assessing the calculation accuracy.

We will show how to formulate the algorithm of approximate calculations based on the operations with functions as well as with formulas.

In the construction of a computational algorithm (13) - (17) we used a priori information available to the task, and first of all information about belonging to a particular class of functions' smoothness which describe the task. Smoothness is determining feature of the diameters' size. The values of the diameters give an idea of the best possible accuracy for the computational algorithm.

Let us introduce Cauchy data as dependent variables

$$\{Y_{p,1}(t), Y_{p,2}(t)\}, p=1, 2m-1. \tag{18}$$

Rewriting SODE (4) as follows

$$Y_{p,n+3}(t) = \frac{Dx1^{2n}}{(n+1)(n+2)} Y'_{p,n+1}(t). \tag{19}$$

From (18), (19) we receive

$$\left\{ \begin{aligned} Y_{p,3}(t) &= \frac{Dx1^2}{2!} Y_{p,1}^{(1)}(t), \\ Y_{p,4}(t) &= \frac{Dx1^2}{3!} Y_{p,2}^{(1)}(t), \\ Y_{p,5}(t) &= \frac{Dx1^4}{4!} Y_{p,1}^{(1)}(t), \\ Y_{p,6}(t) &= \frac{Dx1^6}{6!} Y_{p,2}^{(1)}(t), \\ &\dots \end{aligned} \right. \tag{20}$$

Thus, the general solution of (6) can be represented as follows

$$Y_{p+\varepsilon_x,1}(t, x) = \sum_{n=0}^{\infty} \varepsilon_x^{2n} \cdot \frac{Dx1^{2n}}{(2n)!} \cdot Y_{p,1}^{(n)}(t) + \sum_{n=0}^{\infty} \varepsilon_x^{2n+1} \cdot \frac{Dx1^{2n+1}}{(2n+1)!} \cdot Y_{p,2}^{(n)}(t). \tag{21}$$

The first term of (21) satisfies the adiabatic wall, and the second one satisfies the conditions of the wall at a constant temperature.

In the algebraic area the mathematical model in the form of Cauchy data duplexing

$$\sum_{n=0}^{\infty} Y_{p,1}^{(n)}(t) \cdot \frac{Dx1^{2n}}{(2n)!} = \frac{1}{2} [Y_{p+1,1}(t) + Y_{p-1,1}(t)] \tag{22}$$

$$\sum_{n=0}^{\infty} Y_{p,2}^{(n)}(t) \cdot \frac{Dx^{2n+1}}{(2n+1)!} = \frac{1}{2} [Y_{p+1,2}(t) + Y_{p-1,2}(t)] \quad (23)$$

In general we have the rapid convergence of infinite series (22), (23) at physically realizable of the variables. For example, if the derivatives of $Y_{p,1}(t)$, $Y_{p,2}(t)$ are limited by derivatives of the exponential functions, this is a confirming of term by term differentiation which is used in the analysis. However, in practical cases, the series must converge quickly enough to be able to confine to a few initial terms of the series.

As the aim was to synthesize parallel algorithms of the method with the help of ratio (26) we obtained that the method fits into the concept of unlimited parallelism [2]. Indeed, one processor can be assigned to one node of the design, and it becomes possible to perform calculations on all nodes simultaneously.

Conclusions and prospects for future research. In this paper, an example of solving the problem of Economics and Technology, shows the efficiency of parallelization of dynamical systems. Particular attention is paid to the numerically-analytical methods of solving tasks. Higher speedup compared with the finite-difference approach is explained by the use of analytical solutions that allow you to perform calculations simultaneously and in parallel on all temporary layers without the use of combined memory. This approach excludes recurrent structure calculation of required vectors of solutions which, as a rule, leads to the accumulation of rounding errors. Thus constructed parallel form of the algorithm is maximized and, hence, has the lowest possible time algorithm implementation on parallel computing systems.

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