

# OPTIMAL ADDENDUM COEFFICIENTS OF CYLINDRICAL INVOLUTE GEARING FROM POINT OF VIEW ITS EFFICIENCY

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**Abstract.** *The optimisation of gearboxes, namely of toothed wheels is still valid subject of scientific research. The development of software products for multiparametrical optimisation at Machine Engineering Faculty University of Technology Bratislava was concluded into successful solutions of arbitrary defined objective functions. In the article are given the results of teeth gears optimisation with involute gearing from point of view of its efficiency. The calculation included the complete geometrical and strength check of gearing according to DIN 3990 level B / ISO 6336 as the restriction conditions in original way.*

**Keywords:** *Involute gearing, gearing efficiency,  
DIN 3990 level B / ISO 6336*

**Introduction.** The gearbox is an expensive and technologically demanding product. It significantly affects and very often defines the technical level as well as properties of the final products. Both the conception and basic parameters of gearboxes are decided at their design stage. The quality of this process defines also the a priori operational reliability of the final product and its utility for the final user. It is generally known that the above-mentioned stages of product development affect its quality and technical level up to 90%. So the requirement to improve the initial development stages of gearing by means of suitable optimising methods when designing the main parameters of gearing is obvious.

Prior to the implementation of the optimising process it is necessary to define clearly the main optimisation criterion, the expectations of optimisation of certain crucial parameter and also the simplified suppositions which are not to affect the resulting solution. The problems given remain crucial for the whole design first of all owing to multiformity of structural solutions as well as working loading. Nowadays, there are more approaches to the optimisation tasks in toothed gearing. The common problem of all of them is mainly the effective integration of teeth strength of gearing calculation into the optimisation process. The strength calculation of cylindrical involute gearing is theoretically developed in detail at present. Though in these calculations, just as in most strength calculation of machine parts, only phenomenological approach to dimensioning is applied, regarding to the long-term theoretical research in this field and deluge of experimental measurements, the results obtained this way are reliable mainly in safety check of toothed wheels for fatigue damage by pitting and fracture of tooth dedendum. As for the gearing damage by thermic scoring, both a flash temperature criterion and an integral temperature criterion are used. Both criteria are recommended by ISO and are a part of many standards of strength check of involute gearing. The approach of individual authors to gearing optimisation varies significantly. There are differences in choice of optimisation criterion as well as in their attitude to parameter optimisation in single criteria.

**Experimental.** In the past, one of very frequent criteria of gearing optimisation was a minimum weight criterion. Substantial part of papers on this topic concerns the toothed wheels weight but at the same time did not take into account the fact that considerable part of gearbox weight was just box weight or weight of shafts and bearings. We want to remark that to take note of box weight, shafts and bearings objectively is, because of variety of technical solution, very problematic. Within the increasing requirements on dynamic properties of machines, the criterion of minimum reduced moment of inertia of gearbox is very frequent in literature. In some cases minimum dimension criterion of gearbox (length, width, surface area, build-up volume, etc) are used. The purpose of their use is given in detail in [2]. The problem of gearbox parameter optimisation regarding their production

costs appears sporadically. The reliable solution of this criterion owing to possibilities of getting complex data on production costs and their itemisation from manufacturers is almost impossible. Problems of optimal cylindrical gearing theme regarding its efficiency are not very frequent in literature. Problems of reduction of energy consumption is of high interest and it is the topic of our report.

Gearing efficiency is a criterion in which, from the theoretical point of view, all basic geometric parameters of gearing are directly to effect the value of objective function – efficiency. Limiting requirements of solution are specified by operational condition ( $M_k$ ,  $n$ ,  $N_e$ , oil, temperature), technological indicators (precision of production, surface roughness, materials) and strength requirements under the condition of respecting correct meshing. In the given optimisation calculation we respected the DIN 3990 level B / ISO 6336 for strength requirement. We calculated the coefficient of deprivation (power-loss in gearing)  $H_s$  for straight teeth in dependence on the pitch point position for the momentary value of all task parameters. The coefficient of deprivation  $H_s$  for helical teeth has been calculated by using changing length of contact lines along the engagement. Two types of simplified calculation were used, one by Šalamoun [6]:

$$H_s = \pi \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \frac{\varepsilon_1^2 + \varepsilon_2^2}{\varepsilon_\alpha \cos \beta_b} \quad (1)$$

which is exactly for  $\varepsilon_\beta$  = the whole number and second by Niemann and Winter [7]:

$$H_s = \frac{\pi}{z_1 \cos \beta_b} \left( \frac{u+1}{u} \right) \left( 1 - \varepsilon_\alpha + \varepsilon_1^2 + \varepsilon_2^2 \right) \quad (2)$$

Finally the object function can be then built for one level gearbox defined by axis distance, number of both teeth and wheels and a module, where only addendum modification of pion will be rated for optimisation parameter, using the base formula: where for the centre value of friction coefficient along the pass of contact  $\mu_m$  according to DIN 3990, formula:

$$\eta = 1 - \mu_m H_s \quad (3)$$

Picture 1 presents a progress of functionality gearing efficiency dependence on the addendum modification coefficient of pinion valid for  $z_1 = 19$ ,  $x_c = 0$  and  $u = 1,5$ . It is evident that the objective function has sharp fad – maximum. It can be seen that its position almost does not depend on the definition method of  $H_s$  (in case of leaning teeth).

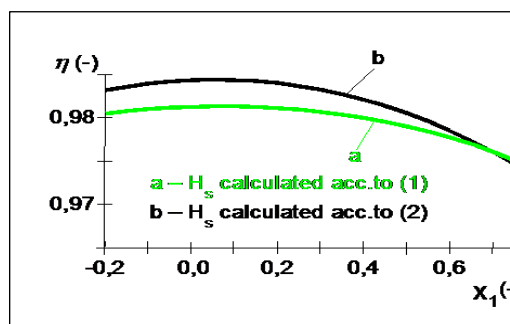


Fig. 1. Dependence of gearing efficiency on addendum coefficient of pinion

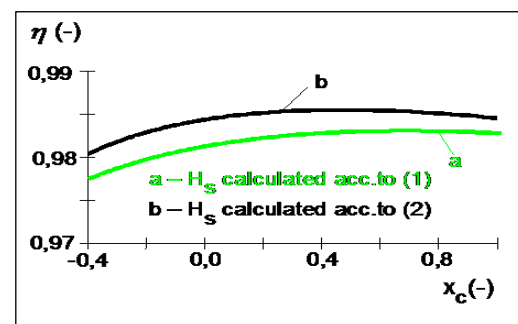


Fig. 2. Efficiency of gearing in dependence on addendum modification-sum of gearing

Fig. 2 gives efficiency value dependence on the whole addendum modification of gearing  $x_c$ . Efficiency advancement is similar in both ways of calculation.

From the advancement of efficiencies in dependence on addendum modification of gearing in Figure 2 fad existence in both ways of calculation is evident, albeit the functional value curve, calculated according to Šalamoun [7] is relatively flat. On the basis of compre-

hensive analysis of this problem we can observe that the fad is really here at lower values of addendum modification of pinion . More interesting and useful for practical application is two-parametrical function (1) optimisation valid for optimisation parameters  $x_1$  and  $x_2$ . The proof that for the target task conceived in this way, the objective function is sharp, is in Figure 3 and Figure 4, they present the values spatial graphs of objective function in dependence on optimisation parameters  $x_1$  and  $x_c$ . The graph in Figure 3 presents calculation results of  $\eta$ , using equation (2), it is obvious that the objective function  $F=f(x_1, x_c)$  has for the maximum efficiency criterion a really sharp extreme. The same is also valid in case of deprivation coefficient calculus according to form (1), it can be seen on the objective function values graphic solution in Figure 4.

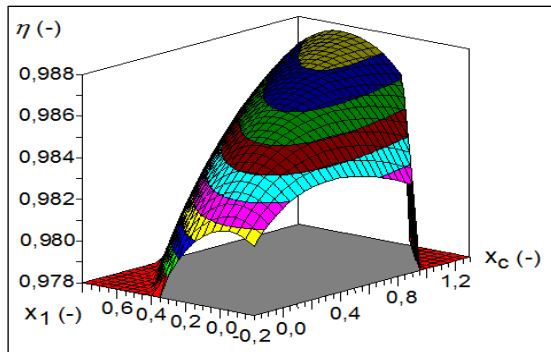


Fig. 3. Simultaneous effect of addendum modification of pinion and addendum modification-sum on gearing efficiency

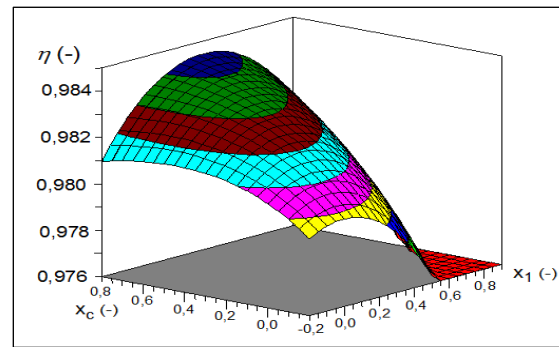


Fig. 4. Simultaneous effect addendum modification of pinion and addendum modification-sum on gearing efficiency

**Results and discussion.** When analysing this two-parameter task it is obvious that the maximum efficiency values can be reached at the maximum practical values of addendum modification of gearing  $x_c$ . Solving the practical tasks, the value of addendum modification of gearing  $x_c$  is usually appointed by desiderative size of centre distance. Then the crucial parameter of present subject function is addendum modification of pinion  $x_1$ . The dependence of optimal values  $x_1$ , for concrete value  $x_c$  in term of efficiency is presented in Fig. 1 – 4.

All calculations were carried out for specific gearing load operations.

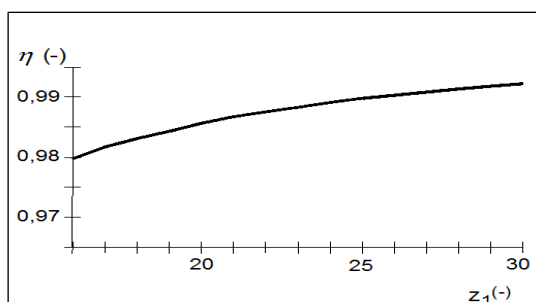


Fig. 5. Dependence of gearing efficiency on pinion teeth number at fixed remaining geometric parameters of gearing

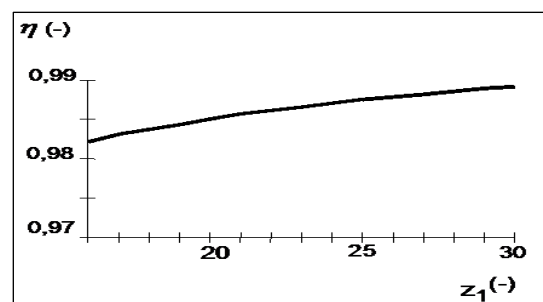


Fig. 6. Dependence of gearing efficiency on pinion teeth number at constant axis distance of gearing (constant gearing load)

The dependence have shown that up to now used teeth modification according to the same values of slide-roll ratios criterion is not optimal. This is valid in general for multi-stage gearboxes, too. The concrete values of gearing ratio allocation in connection with efficiency has its meaning for two-speed gearboxes only and just for lower gear ratio total values. It is recommended to choose for multi-stage gearboxes the gear ratio values of stages for rule:  $u^1 < u^2 < \dots < u^n$  where the last stages gear ratio value is supposed to be near the uppermost limit

of gear ratio values for one stage. The dependence of gearing efficiency in term of geometrical parameters are shown in the fig.5 to 14.

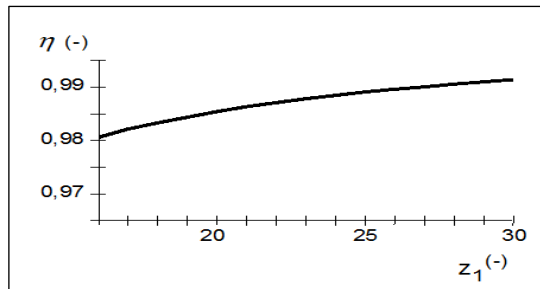


Fig. 7. Dependence of gearing efficiency on pinion teeth number at constant value of transverse gearing load

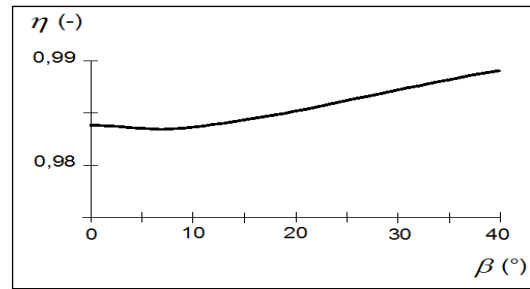


Fig. 8. Dependence of efficiency on teeth helix angle. Efficiency drop at small  $\beta$  angles is caused by the effect of uneven load along gearing represented by coefficient  $K_{H\alpha}$

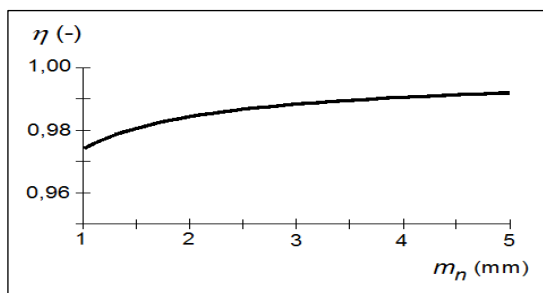


Fig. 9. Dependence of gearing efficiency on modul size at constant number of teeth

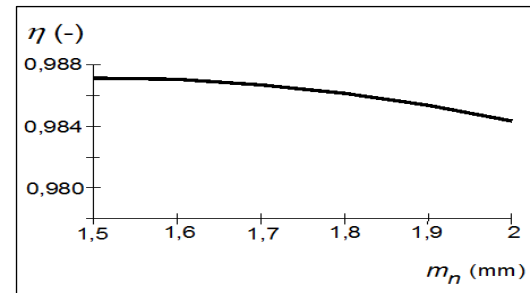


Fig. 10. Dependence of gearing efficiency on modul size at constant values of reference diameters ensured by modification of helix angle

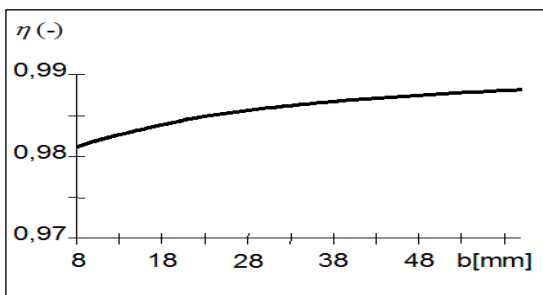


Fig. 11. Dependence of gearing efficiency on pinion width at fixed remaining geometric parameters of gearing

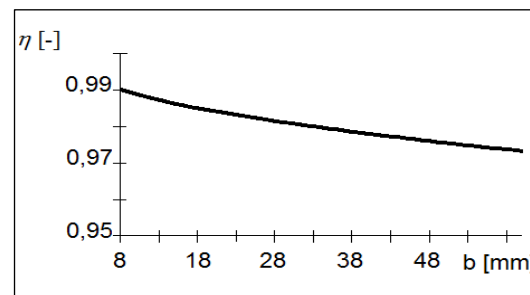


Fig. 12. Dependence of gearing efficiency on pinion width at constant value of transverse gearing load

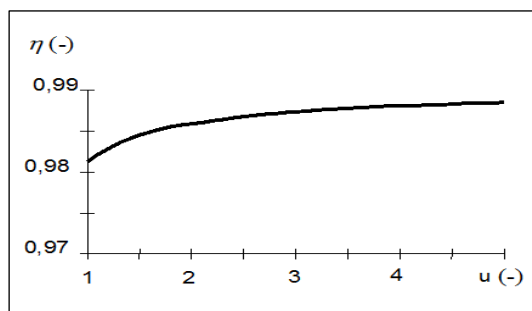


Fig. 13. Dependence of gearing efficiency on the value of gear ratio coefficient

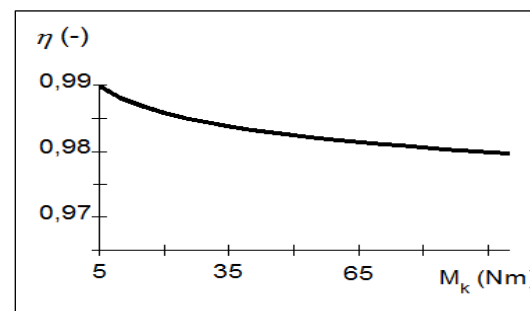


Fig. 14. Dependence of gearing efficiency on torque moment

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