

THE STATE OF THE PRESSED VISCO-PLASTIC MEDIUM OF PLANT-GYPSUM COMPOSITION (PGC) UNDER FLAT DEFORMATION CONDITIONS

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Abstract. It shows the theoretical dependence of the dynamics equations of visco-plastic self-organizing among the masses of plant-weight gypsum located in the plane strain and propose simplified methods for their solutions with close to actual conditions of structure formation of highly porous composition - arbolit

One of the perspective trends in the building complex of the Kyrgyz Republic is using thermal insulation wall materials. In this connection the development of produce of light porous cement wood (arbolit) on the basis of plant-gypsum composition (PGC) from the local agricultural waste (straw) and polymer-silicate gypsum binding material is one of the priorities.

Arbolit as light cement is for wall blocks construction. Developing highly porous composite materials, in particular arbolit on the basis of the plant-gypsum composition materials, they are strengthened by use of quick-hardening gypsum binding materials. Using gypsum instead of cement will allow speed up its hardening without thermal treatment, save fuel and energy resources in building and provide the rapid development of cheap building materials produce for the building complex of the Kyrgyz Republic.

The gypsum binding materials are known to have a number of advantages in comparison with other binding materials: they quickly seal and harden in the open air, have no shrinking contraction and are characterized by sedimentation stability. Moreover gypsum binding materials such as semi-hydrate calcium sulfate is conditioned by the fact that its crystals have needle habitus due to which the strength of highly porous compositions increases.

It is obvious that the strength features of arbolit will depend on the volume concentration of hard, liquid and gas phases. The interaction reaction of the basic components (speed, duration) conditions not only the structural characteristics of the forming plastic mixture but also their impact on the main physic-mechanic characteristics of porous gypsum cement.

Let us analyze the movement of the ideal visco-plastic medium at pressing as a self-organizing porous mass in conditions of flat deformation taking into account the plasticity boundary layers that are components of arbolit composition. We assume that the stressed state of visco-plastic medium if the dynamic coefficient of viscosity μ equals nil and is subject to plasticity condition.

This gives us a possibility under conditions of flat deformation of the moving plastic porous mass to introduce five unknown functions:

$$X_x(x, y, t), \quad Y_y(x, y, t), \quad X_y(x, y, t), \quad V_x(x, y, t), \quad V_y(x_1, y_1, t), \quad (1)$$

- three components of the stress tensor and two projectors of the speed vector on the axle x and y.

Basing on the known equations system that describes the condition of a ideally plastic medium [1] we receive:

$$\frac{1}{\rho_0} \left(\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} \right) + X = \frac{\partial V_x}{\partial t} + v_x \frac{\partial V_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}; \quad (2)$$

$$\frac{1}{\rho_0} \left(\frac{\partial X_y}{\partial x} + \frac{\partial Y_y}{\partial y} \right) + Y = \frac{\partial v_{yx}}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y}; \quad (3)$$

$$(X_x - Y_y)^2 + 4X_y^2; \sin^2 \varphi (x_x - y_y + 2kctg\varphi)^2; \quad (4)$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0; \quad (5)$$

$$\frac{2X_y}{X_x - Y_y} = \frac{(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}) \pm (\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y})tg\varphi}{(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}) \pm (\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x})tg\varphi}. \quad (6)$$

Equations (2) and (3) are equations of the medium movement written by Euler's variables.

ρ_0 – average density of the visco-plastic mass, X, Y – projections of the mass strengths the positive directions of which coincide with the positive axes points.

The equation (4) is a plasticity condition of Tresca Saint Venant and it expresses the constancy condition of the maximum tangential stress which is equal «K».

φ - an angle of the internal friction.

The equation (5) is a condition of the solidness of the moving medium.

The equation (6) expresses the condition of direction coincidence of the maximum deformation speeds of shift with the directions of the sliding lines.

If $\mu \neq 0$ and different from the nil movement speeds in the viscous medium there arises an additional stress system that obeys the law of Newton. It is expressed in the linear scale dependence between the components of the deviator of the additional stress system and the deviator of the deformation speeds:

$$\left. \begin{aligned} X_x^{(2)} &= 2\mu \frac{\partial v_x}{\partial x}; \\ Y_y^{(2)} &= 2\mu \frac{\partial v_y}{\partial y}; \\ X_y^{(2)} &= 2\mu \frac{1}{2} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right), \end{aligned} \right\}; \quad (7)$$

where v – a scale factor that is a function of coordinate and time:

$$p = \frac{X_x + Y_y}{2}.$$

For the unsteady movement of the ideal plastic mass let us introduce four unknown functions χ , β , v_x , v_y , where β - an angle between the positive axes points x and the direction of the main normal stress σ_1 :

$$\chi = \frac{X_x + Y_y}{2K} = \frac{\sigma_1 + \sigma_2}{2K} = \frac{p}{K}, \quad (8)$$

$$v_x = v \cdot \cos\alpha; \quad v_y = v \cdot \sin\alpha, \quad (9)$$

where α - an angle between the positive direction of the x axis and the speed vectors; v – a module of the speed vector.

Correspondingly the stress tensor components receive:

$$\left. \begin{aligned} X_x &= K(\chi + \cos 2\beta) \\ Y_y &= K(\chi - \cos 2\beta) \\ \chi_y &= K \cdot \sin 2\beta \end{aligned} \right\}. \quad (10)$$

Substituting proportions (10) in the equations (2), (3) and (6), we will receive the basic equations system of the unsteady movement of the ideal plastic mass:

$$\left. \begin{aligned} & \frac{\partial \chi}{\partial x} - 2 \left(\sin 2\beta \frac{\partial \beta}{\partial x} - \cos 2\beta \frac{\partial \beta}{\partial y} \right) - \\ & - \frac{\rho_0}{K} \left(\cos \alpha \frac{\partial v}{\partial t} - v \sin \alpha \frac{\partial \alpha}{\partial t} - v^2 \frac{\partial \alpha}{\partial y} \right) + \frac{\rho_0 X}{K} = 0; \\ & \frac{\partial \chi}{\partial y} + 2 \left(\cos 2\beta \frac{\partial \beta}{\partial x} + \sin 2\beta \frac{\partial \beta}{\partial y} \right) - \\ & - \frac{\rho_0}{K} \left(\sin \alpha \frac{\partial v}{\partial t} + v \cos \alpha \frac{\partial \alpha}{\partial t} + v^2 \frac{\partial \alpha}{\partial x} \right) + \frac{\rho_0 Y}{K} = 0; \\ & \left(\cos \alpha \frac{\partial v}{\partial x} + \sin \alpha \frac{\partial v}{\partial y} \right) - v \left(\sin \alpha \frac{\partial \alpha}{\partial x} - \cos \alpha \frac{\partial \alpha}{\partial y} \right) = 0; \\ & \left[\sin(\alpha - 2\beta) \frac{\partial v}{\partial x} + \cos(\alpha - 2\beta) \frac{\partial v}{\partial y} \right] + \\ & + v \left[\cos(\alpha - 2\beta) \frac{\partial \alpha}{\partial x} - \sin(\alpha - 2\beta) \frac{\partial \alpha}{\partial y} \right] = 0. \end{aligned} \right\} \quad (11)$$

In Fig. 1 the mechanical model of the visco-plastic medium is shown. From the above said,

$$\left. \begin{aligned} X_x &= p + K \cdot \cos 2\beta + 2\mu \frac{\partial v_x}{\partial x}; \\ Y_y &= p - K \cdot \cos 2\beta + 2\mu \frac{\partial v_y}{\partial y}; \\ X_y &= K \cdot \sin 2\beta + \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right). \end{aligned} \right\} \quad (12)$$

where p and β – new functions of the stress tensor components.

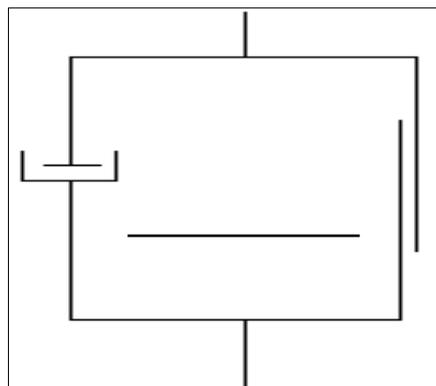


Fig. 1. The mechanical model of the visco-plastic medium

Let us mark the maximum deformation shift speed by H :

$$H = \sqrt{\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2}. \quad (13)$$

Due to the coaxial character of the stress tensor and deformation speeds (6) the proportions (12) can be expressed in the formula:

$$\left. \begin{aligned} X_x &= p + (K + \mu H) \cdot \cos 2\beta; \\ Y_y &= p - (K + \mu H) \cdot \cos 2\beta; \\ X_y &+ (K + \mu H) \cdot \sin 2\beta. \end{aligned} \right\} \quad (14)$$

The proportion (14) equally meets the equation conditions

$$\sqrt{\frac{(X_x - Y_y)^2}{4} + X_y^2} = K + \mu \cdot H. \quad (15)$$

Let us mark by ω – the angle between the axil and direction of the maximum main deformation speed ε_1 . Due to the condition (6)

$$\beta = \omega \pm \frac{\varphi}{2}. \quad (16)$$

On the basis of (16) and [2] we will receive the proportion as follows:

$$\left. \begin{aligned} X_x^{(2)} &= \mu H \cdot \cos 2\omega; \\ Y_y^{(2)} &= -\mu H \cdot \cos 2\omega; \\ X_y^{(2)} &= \mu \cdot H \cdot \sin 2\omega, \end{aligned} \right\} \quad (17)$$

Thus, in the model (picture1) the composing part of the tangential stress on the sliding lines caused by the internal friction is detected by the constituent part of the normal stress $\sigma_n^{(1)}$ and it does not depend on the constituent $\sigma_n^{(2)}$, caused by the strengths of the viscous resistance of the constituent part then the full normal stress is as follows:

$$\sigma_n = \sigma_n^{(1)} + \sigma_n^{(2)}$$

The received equation can be considered as a generalized condition of plasticity of the ideal set visco-plastic medium of the PGC viscous mass.

To receive the simplified equations of dynamics of the viscous plastic mass which are actual in the sphere of the boundary phases layer we will derive the equations as it is done in hydrodynamics.

$$\left. \begin{aligned} \frac{\partial \sigma}{\partial x} - \cos 2\theta \frac{\partial \theta}{\partial x} - \sin 2\theta \frac{\partial \theta}{\partial y} - \\ - \frac{\rho_0}{2K} \left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + \frac{\partial v_x}{\partial t} \right) &= 0; \\ \frac{\partial \sigma}{\partial y} - \sin 2\theta \frac{\partial \theta}{\partial x} + \cos 2\theta \frac{\partial \theta}{\partial y} - \\ - \frac{\rho_0}{2K} \left(v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial t} \right) &= 0; \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0; \\ \sin 2\theta \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) + \cos 2\theta \left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) &= 0, \end{aligned} \right\} \quad (18)$$

где угол θ связан с углом β зависимостью

$$\theta = \beta - \frac{\pi}{4}. \quad (19)$$

Рассматривая какое-либо движение, введем характерные для этого движения некоторую длину L , время T и скорость V . При этом $T = \frac{L}{V}$, а характерным ускорением будет $\frac{V^2}{L}$. Вводим безразмерные величины:

$$\left. \begin{aligned} x &= L \cdot \bar{x}; & y &= L \cdot \bar{y}; \\ t &= T \cdot \bar{t}; & v_x &= V \cdot \bar{v}_x; & v_y &= V \cdot \bar{v}_y. \end{aligned} \right\} \quad (20)$$

Преобразуем уравнение (18) к новым безразмерным переменным. Получим:

$$\left. \begin{aligned} \frac{\partial \sigma}{\partial x} - \text{Cos}2\theta \frac{\partial \theta}{\partial x} - \text{Sin}2\theta \frac{\partial \theta}{\partial y} - \\ - \xi^2 \left(\bar{v}_x \frac{\partial \bar{v}_x}{\partial \bar{x}} + \bar{v}_y \frac{\partial \bar{v}_x}{\partial \bar{y}} + \frac{\partial \bar{v}_x}{\partial \bar{t}} \right) &= 0; \\ \frac{\partial \sigma}{\partial y} - \text{Sin}2\theta \frac{\partial \theta}{\partial x} + \text{Cos}2\theta \frac{\partial \theta}{\partial y} - \\ - \xi^2 \left(\bar{v}_x \frac{\partial \bar{v}_y}{\partial \bar{x}} + \bar{v}_y \frac{\partial \bar{v}_y}{\partial \bar{y}} + \frac{\partial \bar{v}_y}{\partial \bar{t}} \right) &= 0; \\ \frac{\partial \bar{v}_x}{\partial \bar{x}} + \frac{\partial \bar{v}_y}{\partial \bar{y}} &= 0; \\ \text{Sin}2\theta \left(\frac{\partial \bar{v}_x}{\partial \bar{y}} + \frac{\partial \bar{v}_y}{\partial \bar{x}} \right) + \text{Cos}2\theta \left(\frac{\partial \bar{v}_x}{\partial \bar{x}} - \frac{\partial \bar{v}_y}{\partial \bar{y}} \right) &= 0, \end{aligned} \right\} \quad (21)$$

where

$$\xi^2 = \frac{\gamma \cdot \rho_0 \cdot V^2}{2K}, \quad (22)$$

where γ - the coefficient of viscous mass variability, depending on its density.

In the equation (21) non-dimension variables look similar to the dimension ones and instead of the constant $\frac{\rho_0}{2K}$ we can see ξ^2 that tends to infinity. Because of this reason the dash will be omitted in the equation variables and the variable indices will be dropped either. Then the equation system of the boundary layer during the period of the plasticity strength increase at the mass pressing will look like:

$$\left. \begin{aligned} \text{Sin}2\theta \frac{\partial \theta}{\partial y} + \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} &= 0; \\ \frac{\partial \sigma}{\partial y} + \text{Cos}2\theta \frac{\partial \theta}{\partial y} &= 0; \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0; \\ \frac{\partial v_x}{\partial y} &= 0. \end{aligned} \right\} \quad (23)$$

After the equation being integrated (23) the fourth equation is as follows:

$$v_x = f(x, t),$$

where f – an arbitrary integration function.

By substituting in the third equation we will receive:

$$v_y = -\frac{df}{\partial x} \cdot y + g(x, t), \quad (24)$$

where g – an arbitrary integration function.

Substituting v_x and v_y in the first equation (23) we get:

$$\frac{1}{2} \text{Cos}2\theta = \left(\frac{\partial f}{\partial t} + f \frac{\partial f}{\partial x} \right) y - \psi(x, t),$$

And from the second equation we receive:

$$\sigma = -\frac{1}{2} \text{Sin} 2\theta + h(x, t), \quad (25)$$

where ψ and h – arbitrary integration functions.

The arbitrary integration functions f , g , ψ , h allow meeting the boundary conditions on the edges of the mass layer of the PGC during the period of the plasticity strength increase. As the result of transformations the simplified equation of the boundary layer where the plastic dissipation is little is analogical to that in hydrodynamics where the boundary layer in the porous mass contacts with the «body» boundary and with the sphere of the potential current and it looks like:

$$\left. \begin{aligned} V_x &= f(x); \\ V_y &= \frac{\partial f}{\partial x} \cdot y + g(x); \\ \frac{1}{2} \text{Cos}2\theta &= f \cdot y - \psi(x); \\ \sigma &= -\frac{1}{2} \text{Sin} 2\theta + h(x) \end{aligned} \right\}, \quad (26)$$

or

$$\sigma \approx -\sqrt{fy} + \frac{1}{\xi} \cdot \frac{\sqrt{2}}{2} \cdot W(L-x) + \sigma_0, \quad (27)$$

where W – a constant acceleration of the moving mass:

$$W = V_x = \text{const};$$

σ_0 – the integration constant.

Substituting in (27) for σ and omitting little values we will get:

$$\sigma_0 = \frac{1}{2} + \frac{2}{3} \sqrt{aW}, \quad (28)$$

where a – a half of the chosen part length.

The value σ_0 allows calculating the expansion value of the normal pressure P_n , applied to the surface of the plastic «body» mass. This equation equals:

$$\frac{P_n}{K} = 1 + \frac{2}{3} \sqrt{a \cdot W}. \quad (29)$$

So, we have received and analyzed the theoretical dependence of the equations (11, 15, 21, 23, 26, 27, 29) of dynamics of the visco-plastic self-organizing medium mass from the PGC that is under

the flat deformation conditions and have suggested the simplified methods to solve them in close to reality the structure formation of highly porous composition.

Hardening of binding materials (both air so as hydraulic) is of heterogeneous and topo-chemical character that is it goes on the boundary the hard and liquid phases. During the induction period the surface of the boundary is activated and the thermal dynamic and structural conditions are made to form new phases. During this period the physic-mechanical characteristics of the binding composition (viscosity, plasticity strength) at the porous arbolit produce remain almost unchanged and then the strengthening of the structure system takes place. To decrease the induction period it is recommended to use stabilizers, catalyzers, plastificators, polymer- silicate and other supplements.

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