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EKMAN LAYER ROLLS AND THEIR MATHEMATICAL MODELING

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Abstract. *The purpose of this paper is to define and examine the mechanism and possible causes of such type of circulation in the atmospheric boundary layer as rolls. The article is a kind of survey. The history of roll discovery and investigation is briefly described as well as supposed mechanisms of their emergence such as convective, dynamic and parallel instability. Their representative scales and lifetime in the boundary layer are also described. The paper focuses on mathematical description and modeling of circulation by considering the main simplifications and approaches. The attempts to simulate rolls by using the simultaneous solution of systems of equations of the mean flow and rolls are described. The modified profiles of wind speed that take into account the presence of helicity in the boundary layer are used for modeling of the mean flow. The inclusion of helicity is appeared to make a significant contribution in the simulation of boundary layer processes. Roll simulation over the Black sea coast using the model of regional atmospheric circulation Advanced Research WRF 3.3.1 is also presented. Using different parameterizations allows through numerical experiments to consider dynamic processes occurring on the border of the "land – sea" and to examine the contribution of hydrodynamic instabilities of various types of roll formation.*

Keywords: *atmospheric circulation rolls, mathematical modeling of Ekman boundary layer, cloud streets, helicity.*

Indroduction

In the late 1960s the assumptions about the presence of an orderly circulation in the Atmospheric Boundary Layer (ABL) appeared, and were confirmed by numerous experiments in the next two decades. It is noteworthy that for the first time such type of the circulation was investigated by an American chemist I. Langmuir in 1938 for the surface layer of lakes and seas. He noticed the wind streaks on the surface were usually built in the direction of the wind nearly. They can be easily recognized because they accrue spindrift, algae and plankton. As a result of field experiments on the Lake George (the Empire State) Langmuir concluded that the wind bands mark the circulation in the surface layer of water that represents a range of oppositely directed vortices. The band is formed in the convergence zone at the surface of the water. Still a general theory that is able to explain the mechanism for orderly circulation does not exists. In 1963 Professor at the University of Maryland A.Fuller suggested that its cause is the shear nature of Ekman spiral of distribution of wind speed presented both in the ocean and the atmosphere. Nowadays the instability of Ekman flow is also considered to be one of the main reasons for the formation of the quasi-two-dimensional ordered vortices in the ABL. [1]

Description of the phenomenon

The circulation considered is a system of elongated vortices rotating in the horizontal direction. The axis of rotation may coincide in its direction with a mean flow in the ABL or be deflected from it not more than 30˚, the angle of rotating being affected by the thermal stratification in the ABL. The figure 1 depicts a general scheme of the circulation. It's important to understand, that clouds originating in the regions of divergence with updrafts are only «indicators» of existence of this phenomenon. Because of the appearance on satellite images it is known as «cloud streets» or «rolls» in scientific literature (fig.2). This circulation can be detected on satellite synthetic aperture radar images in the form of linear strips with parameters close to ones of rolls under conditions that not conducive to cloud formation. In such case it may found a term "thermal streets" in the literature.

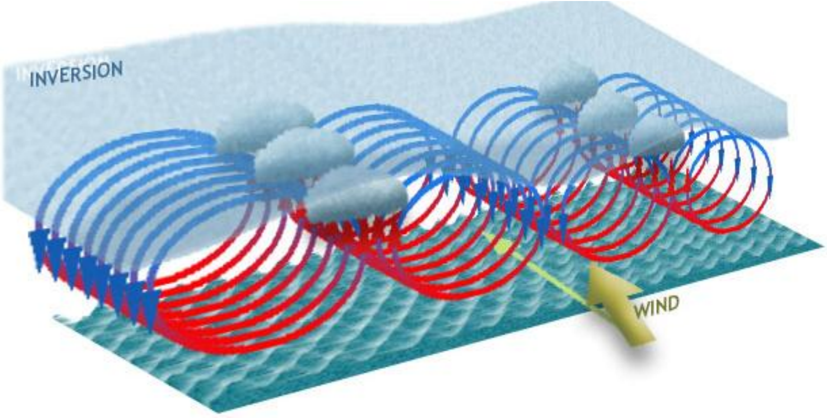

Fig.1. General mechanism of the circulation

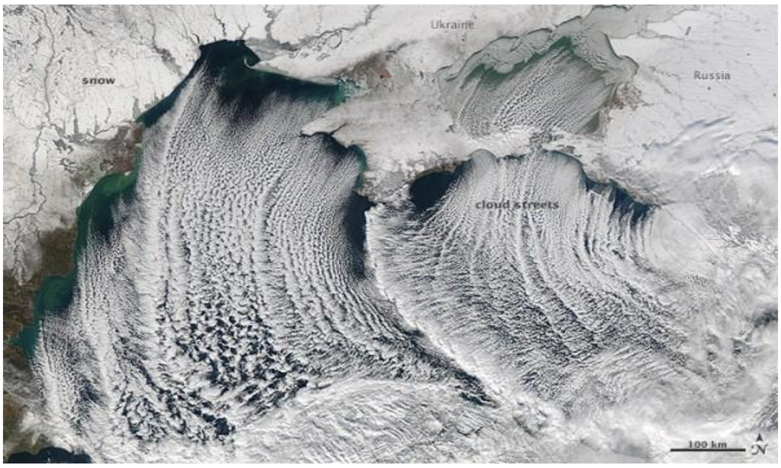

Fig.2. A satellite image over the Black sea. January 2015.

The representative scales of rolls are such that intercloud distance is from 2 to 8 km, extension is from 20 to 500 km, and depth is within the ABL. Rolls are often formed at a distance of about 100 km from the coastline during cold-air outbreaks over the warm open water surface. The velocity of updrafts and downdrafts may reach 2 m/sec. The average lifetime of these structures is about a day. [2]

The current experimental data suggest that rolls play an important role in the dynamics of the ABL. They provide up to 60% of heat and mass transfer, store a significant part of the turbulent energy and influence the wind and temperature profiles of the mean flow. Their contribution to the vertical transport was found to dominate over turbulent one in high latitudes. Furthermore, now they should be taken into account while analyzing and interpreting the ABL characteristics because the occurrence zones of convergence and divergence in horizontally homogeneous mean flow as a result of rolls' circulation leads to areas of high humidity, turbulence or concentration of impurities. Thus the passive diffusion of impurities model becomes unacceptable. [3]

The emergence of rolls is associated with hydrodynamic instabilities of three types such as convective, dynamic and parallel instabilities, the first two types contributing most to development of rolls. The traditional mechanism is that initial convective cells are rebuilt by the wind in the longitudinal eddies, but many details are not clear yet. [4]

Mathematical modeling of rolls

In the 1960s-1980s rolls have been studied mainly by using different experimental methods. The flight benches, high-altitude meteorological masts, radiosonde measurements and satellite observations were used. Now the predominant way of study is mathematical modeling because of the difficulty of obtaining necessary data with high spatial resolution, and besides it allows a better understanding of interaction between rolls and external environment by using numerical experiments. Also only since the late 1980s it became possible to achieve the desired resolution of the models to describe rolls and to implement a quite complicated numerical scheme to study the instability of stratified flow.

The modeling of rolls has certain peculiarities and simplification. The simplest models consider their formation in the horizontally-homogeneous stratified thermally ABL. It is believed that rolls are the consequence of loss of laminar flow stability, so the mathematical description uses the theory of hydrodynamic stability. The mean values and ordered components of the fluctuating part of the flow are considered separately. The equations describing the behavior of various components of motion are obtained from the Navier-Stokes equations for an incompressible viscous fluid in a rotating coordinate system in the Boussinesq approximation. Rolls' characteristics are assumed to be homogeneous along their axis, so their parameters are usually considered in the cross section. The boundary conditions at the upper boundary are used because turbulence in the free atmosphere is weak and not linked to the processes in the ABL, and at the lower boundary their formulation depends on whether the model simulates the lowest atmospheric layer separately. There are a number of models considering separately the core of the ABL (where rolls are formed) and the surface layer being described in accordance with the Monin - Obukhov theory. Therefore it is necessary to formulate the lower boundary condition in such a way as to take into account the transition from the surface layer to the ABL. In addition to equations of motion for the mean and ordered components and boundary conditions standard models include formulas of closure for turbulence characteristics and correlation for some empirical coefficients. The system of equations is non-dimensionalized and it needs certain external and dimensionless parameters before being implemented. [5]

Nowadays it's important to introduce the helicity presented both in rolls and Ekman flow. The Earth's rotation and surface friction lead to large-scale geostrophic wind turning with the height. It's Ekman spiral. This is the source of the helicity of the turbulent component, which results in changing the structure of the tensor of turbulent Reynolds stresses directly affecting the mean flow. According to the results of numerical simulations the introducing of helicity in the ABL significantly reduces averange energy of turbulent motion, turbulent viscosity and the rotation angle of the Ekman spiral (that was assumed to allow further developments of K-theory previously) and increases the effective thickness of the boundary layer. Convection enhances the influence of helicity. [3]

One of the latest overall models (Ponomarev V.M., Ckhetiani O.G., Shestakova L.V., 2009) of interaction between rolls and mean flow in non-dimensional form is the following:

- rolls' equations (1) – (7)

$$Re\cdot\left(\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}+\frac{\partial \psi}{\partial y}\cdot\frac{\partial U}{\partial z}-\frac{\partial \psi}{\partial z}\cdot\frac{\partial u}{\partial y}+\frac{\partial \psi}{\partial y}\cdot\frac{\partial u}{\partial z}-\langle\frac{\partial \psi}{\partial y}\cdot\frac{\partial u}{\partial z}\rangle+\langle\frac{\partial \psi}{\partial z}\cdot\frac{\partial u}{\partial y}\rangle\right)=\Delta u-g\frac{\partial \varphi}{\partial z}-2\frac{\partial \psi}{\partial z},$$

$$Re\cdot\left(\frac{\partial \varphi}{\partial t}+V\frac{\partial \varphi}{\partial y}+\frac{\partial \psi}{\partial y}\cdot\frac{\partial^2 V}{\partial z^2}-\frac{\partial \psi}{\partial z}\cdot\frac{\partial \varphi}{\partial y}+\frac{\partial \psi}{\partial y}\cdot\frac{\partial \varphi}{\partial z}-\langle\frac{\partial \psi}{\partial y}\cdot\frac{\partial \varphi}{\partial z}\rangle+\langle\frac{\partial \psi}{\partial z}\cdot\frac{\partial \varphi}{\partial y}\rangle+Ri\cdot\frac{\partial \theta}{\partial y}\right)=\Delta\varphi+ g\frac{\partial}{\partial z}\Delta u-2\frac{\partial u}{\partial z},$$

$$Pr\cdot Ri\cdot\left(\frac{\partial \theta}{\partial t}+V\frac{\partial \theta}{\partial y}+\frac{\partial \psi}{\partial y}\cdot\frac{\partial T}{\partial z}+\frac{\partial \psi}{\partial y}\cdot\frac{\partial \theta}{\partial z}-\frac{\partial \psi}{\partial z}\cdot\frac{\partial \theta}{\partial y}-\langle\frac{\partial \psi}{\partial y}\cdot\frac{\partial \theta}{\partial z}\rangle+\langle\frac{\partial \psi}{\partial z}\cdot\frac{\partial \theta}{\partial y}\rangle\right)=\Delta\theta,$$

$$\varphi=-\Delta\psi.$$

- the horizontally-homogenous ABL's equations including thermal stratification (5) – (7)

$$Re\cdot\left(\frac{\partial U}{\partial t}-\langle\frac{\partial \psi}{\partial z}\cdot\frac{\partial u}{\partial y}\rangle+\langle\frac{\partial \psi}{\partial y}\cdot\frac{\partial u}{\partial z}\rangle\right)=\frac{\partial^2 U}{\partial z^2}-g\frac{\partial^2 V}{\partial z^2}+2\cdot\left(V-V(z_\infty)\right),$$

$$Re\cdot\left(\frac{\partial V}{\partial T}-\langle\frac{\partial \psi}{\partial z}\cdot\frac{\partial^2 \psi}{\partial y\partial z}\rangle+\langle\frac{\partial \psi}{\partial y}\cdot\frac{\partial^2 \psi}{\partial z^2}\rangle\right)=\frac{\partial^2 V}{\partial z^2}+g\frac{\partial^2 U}{\partial z^2}+2\cdot\left(U-U(z_\infty)\right),$$

$$Re\cdot Pr\cdot\left(\frac{\partial T}{\partial t}-\langle\frac{\partial \psi}{\partial z}\cdot\frac{\partial \theta}{\partial y}\rangle+\langle\frac{\partial \psi}{\partial y}\cdot\frac{\partial \theta}{\partial z}\rangle\right)=\frac{\partial^2 T}{\partial z^2}.$$

- modified wind profiles' equations (8) – (9)

$$U=\cos(\varepsilon)-exp(-z_\infty)\cdot\cos(g' z_\infty+\varepsilon),$$

$$V=-\sin(\varepsilon)+exp(-z_\infty)\cdot\sin(g' z_\infty+\varepsilon),$$

where *u,v,w* are rolls' velocity components*; U,V* are the mean flow's velocity components ; ψ is the stream function*;* φ is the projection of the vorticity on the *X*-axis*; T* is the mean flow's temperature*;* is rolls 'temperature*;* $\langle\rangle$ is the operator of averaging over the coordinate*;* g' is the parameter of helicity; ε is the deflection angle of rolls relative geostrofic wind *; Re, Ri, Pr* are the numbers of Reynolds, Richardson and Prandtl respectively; Z_∞ is the upper boundary of the computational domain.

The modified version of the hypothesis of eddy viscosity that takes into account the influence of helicity on turbulent structure stresses is used to calculate the helicity. Boundary conditions for equations describing rolls' structure are the following: (a no-slip condition on the lower boundary), (no friction at the upper boundary). The temperature of rolls Θ equals to zero both at the upper and lower boundaries. Periodic boundary conditions are stated in the direction that is perpendicular to rolls' axis of rotation. More accurate models take into account the correct transition from the surface layer to the ABL also.

The lower boundary conditions for the system of equations of the mean flow take the form $U(0) = V(0) = 0$ and $T(0) = Ts$, where Ts is the temperature of surface, the upper boundary ones for wind using the values calculated directly by equations (8) – (9), the temperature of the mean flow on the top boundary having the condition $T=Ts\pm T$, where T is the difference of temperature on upper and lower boundaries.

The parabolic transport equations (1) - (3) and the elliptic Poisson's equation (4) are solved separately in the numerical realization, the former being solved with the introduction of model transport equations.

The implementation of the model allows to obtain a periodic structure of rolls' fields of longitudinal velocity component u, stream function ψ, vorticity φ and temperature Θ for a given values of the angle of rotation of rolls relative to the mean flow ε, Reinolds and Richardson numbers (Re and Ri respectively), the parameter of helicity g' and temperature difference at the upper and lower boundaries. It's possible to follow the evolution of rolls, the dynamics of their energy and helicity. The results of the numerical experiment are consistent with field observations and theoretical ideas. The authors of the model suggest that it could be the basis of constructing two-level eddy simulation models. [4]

This phenomenon is well marked on the shores of the Pacific and Arctic oceans, and the Caspian and the Black sea in Russia. The numerical simulation of rolls of the Black sea coast was performed by using the regional atmospheric circulation Advanced Research WRF 3.3.1 model on three nested domains 9*9, 3*3 and 1*1 km with 37 levels in the vertical with increased resolution in the ABL. It was supposed to simulate and study small-scale structure of the vortex formed on 15 August 2007 near the coast of the Crimean Peninsula. The following parameterizations were used: Rapid Radiative Transfer Model (RRTM) and the Dudhia scheme for the calculation of the radiation balance longwave and shortwave radiation respectively, scheme Kain–Fritch – for the calculation of cumulus convection in domains with a resolution of 9 and 3 km (it were calculated explicitly and parameterization was not needed in the domain with 1 km resolution). The scheme of the Single-Moment 3-class was applied to describe phase transitions in the atmosphere, and the scheme MM5 similarity was used for the parameterization of friction of the surface layer. Planetary boundary layer was parametrisable using the Yonsei University scheme which in the coefficient of vertical eddy viscosity Kz is set in the form of linear-parabolic profile $Kz = \kappa wz(1 - z/H)2$, where κ=0,4 is the Karman constant; w is the vertical scale of wind speed; H is the thickness of the boundary layer. Input data for an external domain was the data analysis of the operational Global Final Analyses (FNL) with a resolution of 0.5 × 0.5 deg which was updated every 6 h. After adaptation of the model to specified initial conditions the development of atmospheric processes in all three domains is determined only periodically changing boundary conditions on the external domain.

The purpose of the study was not only the reproduction of characteristics close to those actually observed, but also conducting numerical experiments to better understand the contribution of hydrodynamic instabilities to the development of this circulation. So, it was assumed that advection in a background flux of local inhomogeneities of the velocity field (that was inevitable on the border of land and sea) contributed in addition to convective and dynamic instabilities. Its influence is expressed mathematically by Okubo-Weiss criterion. The main reason for the formation of rolls was supposed to be the convection in the ABL over the sea. To identify the role of the convective instability the ratio of the inversion height Zi to the Monin – Obukhov parameter Lmo was used instead of Rayleigh number, which allowed to determine the input of buoyancy forces into the generation of turbulent kinetic energy in unstable stratified shear flow. A numerical experiment where convective instability was synthetically suppressed, but the influence of advection and deformation in the velocity field of the evolution of inhomogeneities carried on was made to highlight the mechanism of occurrence of rolls. There were two numerical experiments with decreasing sea surface temperature (SST) firstly at 5°C and then at 10°C, all other conditions being unchanged. The similar decrease of SST was assumed to repress convective instability and highlight possible additional causes of the appearance of rolls.

According to the experiment results the fall of the SST by 5° C and then 10° C reduced the life time of the Crimean vortex (~ 20 and 30% respectively), but did not change significantly its large-scale structure. The height, radius and orbital velocity of the vortex in both experiments changed slightly. The height of the ABL significantly decreased as had been expected. The dynamic instability, which was considered as another possible reason for the appearance of the circulation, did not develop because necessary condition for the formation of this type of instability such as the presence of inflection points on vertical profiles of horizontal component of speed of background flow was not performed. The appearance of rolls has also been associated with the advection of the velocity field inhomogeneities. [6]

Conclusions

The physically based parameterization of rolls is particularly important for the prediction of transport processes in the ABL. However, there are still many questions that need competent physical and mathematical description, such as the occurrence of this circulation, the instability of mean flow, the nature and dynamics of helicity. Many models underestimate the speed of circulation in rolls by 30-40%. There are few works modeling non-stationary problems as well as giving consistent results in the range of unstable stratification. [3] A number of similar problems can be solved by numerical simulation experiments.

REFERENCES

1. Ryanzhin S., Kochkov N., Karlin L. Misterious circulations. Tne nature. 2008. №4.
2. Mikhailova L., Ordanovich A. Coherent structures in the atmospheric boundary layer. Izvestiya of Academy of sciences, USSR. Atmospheric and oceanic phisics. 1991. Vol.27. №6. P. 593 – 613.
3. Ponomarev V., Khapaev A., Chkhetiani O. Role of helicity in the formation of secondary structures in the Ekman Boundary Layer. Izvestiya of Russian Academy of sciences. Atmospheric and oceanic phisics. 2003. Vol. 39. №4. P. 435 – 444.
4. Ponomarev V., Chkhetiani O., Shestakova L. Numerical modeling of the developed horizontal circulation in the atmospheric boundary layer. Computational mechanics of continuous medium. 2009. Vol.2. №1. P. 68 – 80.
5. Modeling two-dimensional rools in the atmospheric boundary layer. Meteorology and hydrology. 1988. №11. P. 29 – 42.
6. Yarovaya D., Efimov V. Numerical modeling of rolls in mesoscale vortices over the Black Sea. Nautical hydrophysical journal. 2013. №4. P. 61 – 72.