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JOURNAL	World Science
p-ISSN	2413-1032
e-ISSN	2414-6404
PUBLISHER	RS Global Sp. z O.O., Poland
ARTICLE TITLE	DETERMINING HEAT LOSSES FROM THE BUILDING ENVELOPE USING THE NON-STATIONARY METHOD
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ARTICLE INFO	Nodar KEVKHISHVILI, Tengiz JISHKARIANI, Nikoloz JAVSHANASHVILI. (2023) Determining Heat Losses from the Building Envelope Using the Non-Stationary Method. <i>World Science</i> . 3(81). doi: 10.31435/rsglobal_ws/30092023/8056
DOI	https://doi.org/10.31435/rsglobal_ws/30092023/8056
RECEIVED	26 August 2023
ACCEPTED	29 September 2023
PUBLISHED	30 September 2023
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DETERMINING HEAT LOSSES FROM THE BUILDING ENVELOPE USING THE NON-STATIONARY METHOD

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DOI: https://doi.org/10.31435/rsglobal_ws/30092023/8056

ARTICLE INFO

ABSTRACT

Received: 26 August 2023 Accepted: 29 September 2023 Published: 30 September 2023

KEYWORDS

Climate Change, Natural Gas, Carbon Dioxide Emission, Heat Exchange

One of the main causes of the climate change is accumulation of huge amount of carbon dioxide (CO₂) in the atmosphere emitted from the combusting of organic fuels (coal, oil products and natural gas), consequently, to slow down the progress of the global warming is directly related to the limitation of CO₂ emission which could be achieved through the rational use of fuel and energy in every sector (industrial, household, transport and building sectors), introduction of energy-saving measures including highly efficient technologies and innovative methods. The building sector accounts for about 40% of the energy saving potential, therefore reduction of energy losses is the best way to reduce energy consumption of buildings. To calculate the heat loss from the building envelope, it is necessary to know the thermal conductivity coefficient (λ) of each construction element. Currently developed methods of λ determination are entirely based on the laboratory test using the stationary regime. For more realistic results, it is necessary to take into account the daily variability of temperature and non-stationary thermal conductivity processes. Solving the non-stationary thermal conductivity tasks are associated with the significant difficulties due to the application of the relatively complex mathematical equations. Usually, the theory of non-stationary thermal conductivity refers to the method of separation of variables or the so-called Laplace Transform, which requires the use of operational counting methods. The article presents an innovative method for determining the coefficient of thermal conductivity (λ) of each construction element in the nonstationary temperature regime, which enables determination of heat losses from the building envelope in real environment using the precise definition of thermal flow velocity.

Citation: Nodar KEVKHISHVILI, Tengiz JISHKARIANI, Nikoloz JAVSHANASHVILI. (2023) Determining Heat Losses from the Building Envelope Using the Non-Stationary Method. *World Science*. 3(81). doi: 10.31435/rsglobal_ws/30092023/8056

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INTRODUCTION.

Non-stationary thermal conduction processes are widely used in a variety of machinery and equipment in different industries where the temperature undergoes periodic variations. To ensure their reliable operation, the temperatures of individual structural elements must be determined in advance at each moment of time, in order to specify how long it will take for the temperature to reach the required value in the defining area of this element, after a certain temperature environment is created around it. It is necessary to take into account the 24 hours variation of temperature and implement non-stationary processes of heat conduction, as well as when determining the temperature regime in buildings, determining heat losses from the building envelope, implementing energy-saving measures and developing a corresponding strategy. Solving problems of nonstationary heat conduction requires the use of a relatively complex mathematical equations and is therefore associated with significant difficulties. Usually, in the theory of non-stationary heat conduction, the method of separation of variables or the so-called Laplace transformation is applied, which involves the use of operational accounting.

MAIN PART.

To determine the heat flow velocity in a non-stationary temperature regime, consider an unbounded flat wall¹, the temperature distribution along the thickness of which corresponds to the function $t(x)|_{\tau=0} = F_x = t_0$. At the initial moment, the temperature of the side surfaces of the wall suddenly becomes equal to t_c , which is constantly maintained throughout the subsequent heat exchange process. We need to find the temperature distribution in the thickness of the wall.

For the task under consideration, it is convenient to place the coordinate system in the center of the wall (Fig. 1).



Fig. 1. Heat exchange diagram of an unbounded wall.

The condition of the problem can be formulated as follows: the differential equation of thermal conductivity is given:

$$\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t(x, \tau)}{\partial x^2}, \quad (\tau > 0, -\delta \le x \le \delta)$$
(1)

with boundary conditions:

$t(x, 0) = F(x) = t_0$,	$\tau = 0$	(2)
$t(+\delta, \tau) = t_c = \text{const},$	$x = +\delta$	(3)
$t(-\delta, \tau) = t_c = const$,	$x=-\delta$	(4)
$\frac{\partial t}{\partial x} = 0 \qquad x = 0$		(5)

Where, $a = \lambda / \rho c$.

Using the Fourier method, the specific solution of the differential equation (1) will take the form:

 $\frac{\partial t}{\partial z} = 0$).

¹ An unbounded wall is a wall whose length and width are infinitely large compared to the thickness, and the temperature changes only in the *x* direction, and is constant in the *y* and *z* directions. ($\frac{\partial t}{\partial y} = 0 \mod x$

$$t(x,\tau) = \sum_{i=1}^{\infty} D_n e^{-a k_n^2 \tau} \cos(k_n x) \qquad (6)$$

The parameter k is determined from the equation:

$$k_n \delta = (2n-1)\frac{\pi}{2}, \ n = 0, 1, 2, 3...$$
 (7)

and Dn coefficient is calculated from the Fourier integral:

$$D_n = \frac{2}{\delta} \int_0^{\delta} t_0 \cos(k_n x) \, dx = \frac{2t_0}{k_n \cdot \delta} \sin(k_n \delta) \quad (8)$$

By entering the value of Dn coefficiens in (6), we will finally have:

$$\frac{t(x,\tau)-t_c}{t_0-t_c} = 2\sum_{n=1}^{\infty} \frac{\sin(k_n\delta)}{k_n\delta} \cos(k_nx) e^{-ak \frac{2}{n}\tau}$$
(9)

Equation (9) determines the temperature regime and, accordingly, the speed of heat flow velocity in a flat wall when one side of it has a constant temperature [2].

In order to estimate the velocity of heat flow from a partially heated wall to the other surface of the wall, it is necessary to uniquely determine the minimum radius of the heated circular area, where the current lines of the heat flow in the central part will remain parallel to each other. That is, it will be possible to use equation (9) of the one-dimensional heat flow process.

For this, consider a body bounded on one side, the temperature in the entire volume of which is uniform at the initial moment of time. The entire surface of the body, except for the circle with radius R, is covered with insulation. Starting from a certain moment of time, the temperature on the entire area of the circle becomes constant equal to ϑ_0 , it is clear that the heat from this circular area will gradually spread throughout the depth of the body. It is necessary to determine the stationary distribution of the temperature flow.

Fig. 2 shows the cylindrical coordinate system r, φ , z, which is located in such a way that the positive side of the z axis is directed from the circular area to the depth of the body. Obviously, with such an arrangement of the coordinate system, the temperature flow will no longer depend on the φ coordinate.



Fig. 2. Stationary temperature field in a body bounded on one side.

In this case, the differential equation of heat conduction in cylindrical coordinates and the boundary conditions are written in the following form:

$$\frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \vartheta}{\partial r} + \frac{\partial^2 \vartheta}{\partial z^2} = 0 \tag{10}$$

Boundary conditions:

$$\vartheta = \vartheta_0 = \text{const}, \ z = 0, \ o \le r \le R$$

$$\vartheta = 0, \ z = 0, \ r > R$$

$$\vartheta = 0, \ z \to \infty, \ o \le r < \infty$$

$$\vartheta = 0, \ r \to \infty, \ o < z < \infty$$

$$(11)$$

Using the method of separation of variables, the specific solution of equation (10) will take the following form:

$$\vartheta = c e^{-k z} \Psi(r) \tag{15}$$

where, $\Psi(\mathbf{r})$ is the solution of Bessel's differential equation:

$$\mathbf{r} \, \Psi^{"} + \Psi^{"} + \mathbf{k}^2 \mathbf{r} \, \Psi = \mathbf{0}$$

The general solution of the Bessel equation is defined as the sum of zero-order Bessel functions of the first Jo (kr) and the second Yo (kr):

$$\Psi(\mathbf{r}) = C J_{o}(\mathbf{kr}) + D Y_{o}(\mathbf{kr})$$
(16)

In the general solution, the coefficient D should be equal to zero, because the function Yo (kr) when r=0 tends to infinity, which contradicts the boundary condition (11). Thus, the general solution of the thermal conductivity equation (10) will be written in the following form:

$$\vartheta = \sum_{n=1}^{\infty} C_n e^{-k_n z} J_o(k_n r) \qquad (17)$$

Since the problem is given by boundary conditions of the first and not the third kind, and the body is unbounded in the positive direction of the z axis, there are no constraints on the parameter k, written as a transcendental equation. This means that k can pass through a continuous sequence of numbers, e. i. Its two closest values differ from each other by an infinitesimally small amount dk. In addition, we can replace the constant C of integration with some function f(k), then (17) the infinite row passes into the integral and the general solution will be:

$$\vartheta = \int_{0}^{\infty} f(k) e^{-k z} J_{0}(kr) dk \qquad (18)$$

where, the form of the function f(k) is determined from the boundary conditions. (11) in the boundary condition it is required that, when $o \le r=R$ an

$$\vartheta|_{z=0} = \int_{0}^{\infty} f(k) J_{0}(kr) dk$$

to take a constant ϑ o value. In the area r > R, the same condition does not introduce any restrictions. On the other hand, it is clear that the function cannot maintain a constant ϑ o value, because according to condition (13) it must tend to zero as r increases.

(12) the boundary condition leads to a similar result. According to this condition, the integral:

$$\frac{\partial \vartheta}{\partial z}\Big|_{z=0} = -\int_{0}^{\infty} k f(k) J_{0}(kr) dk$$
(19)

When r > R, it must be equal to zero, and in the area $o \le r \le R$ its value can be any. The only restriction is that the integral (19) should not take a value equal to zero, because in this case there will be no place for the flow in the body.

Non-proprietary, parameter-dependent integrals are known from mathematics [3], which have the feature that the character of the functional dependence on any value of the parameter changes suddenly.

The most famous integral

$$\int_{0}^{\infty} \frac{\sin(mR)}{m} \cos(mr) \, dm = \begin{cases} \pi/2, & r < R \\ \pi/4, & r = R \\ 0, & r > R \end{cases}$$

The nature of the change of this integral depending on the parameter R is shown in Fig. 3.



Fig. 3 Parameter-dependent non-proprietary integrals.

Integrals with a similar property can be represented using Bessel functions [2], in particular, the following integrals (Fig. 3 b, c):

$$\int_{0}^{\infty} \frac{\sin(mR)}{m} J_{0}(mr)dm = \begin{cases} \frac{\pi}{2}, & o < r \le R\\ \arcsin\left(\frac{R}{r}\right), & r > R \end{cases}$$
(20)
$$\int_{0}^{\infty} \sin(mR) J_{0}(m \cdot r) dm = \begin{cases} \frac{1}{\sqrt{R^{2} - r^{2}}}, & o < r \le R\\ o, & r > R \end{cases}$$
(21)

Fig. 3. It can be seen from b, and c that integral (21) satisfies condition (12), and integral (20) satisfies condition (11), if we include in it such a quantity that instead of $\pi/2$ will give us ϑ o. Thus, if we assume in integral (18) that

$$f_k = \frac{2}{\pi} \vartheta_o \frac{\sin(KR)}{K}$$

Then the equation of the temperature regime to be searched will take the following form:

$$\vartheta = \vartheta_0 \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin(KR)}{K} J_0(kr) e^{-k \cdot z} dk$$
 (22)

Thus, equation (22) can be used to determine the radius of the minimum heating area R on one side of the wall, which ensures the possibility of using equation (9).

In order to determine the λ coefficient of thermal conductivity in a non-stationary temperature regime, a special experimental device was created by the team of professors of the "Scientific-Engineering Center of High-Temperature Thermophysical Processes" in the Faculty of Energy at the Georgian Technical University, which provides the accurate assessment of the thermal characteristics of buildings and energy losses. It will be possible to use it both in the teaching laboratory for educational purposes, and on-site conditions to make real measurements and record relevant data.

Below is the device for measuring the coefficient of thermal conductivity and the sample of the material to be measured.



Fig. 4. Device for measuring thermal conductivity coefficient.

Steel grades SG 20 and 45G2 with the known thermal conductivity were initially used to test the thermal conductivity measuring method presented above (Table 1):

Steel	$\lambda [W/m \cdot K]$	Steel Grade	$\lambda [W/m \cdot K]$	Steel Grade	$\lambda [W/m \cdot K]$
Grade					
1	2	3	4	5	6
10	69.5	65	58	30 XMA	30.6
15	66.5	70	58	35 XM	35.7
20	67	30 G	64.5	40 XΦA	45
25	67	40 G	51	12 XH2	20.6
30	65	50 G	33	12 XH3A	-
35	65	30 X	39.6	30 XFC	32.4

Table 1. Steels with known thermal conductivity SG 20 and 45G2.

1	2	3	4	5	6
40	57	35 X	36.5	1X 13	-
45	58.6	35 G2	-	2X 13	25
50	58.5	40 G2	-	3X 13	17.8
55	58.5	45 G2	38.2	X18H9	14.6
60	58	50 G2	34.7	X18H9T	13.4

Table 1. Continuation.

Thermal conductivity in metals significantly depends on the alloying quality of the material. In pure and low-alloyed materials, its value is relatively higher than in alloyed ones [4; 5].

The diameter of the measured materials was 50 mm, and the length was 90 mm. As required by the methodology, it was thermally insulated from three sides, and one end surface maintained a temperature of 40 $^{\circ}$ C during the experiment. The time of the heat flow reaching from one surface to the other was 27 seconds and 47 seconds, respectively.



Fig. 5. Temperature distribution in steels: SG 20 and 45G2.

From the comparison of theoretical and experimental results, it can be seen that in metallic materials, the error of thermal conductivity coefficients measured by the presented method does not exceed 1.6%.

In further research, 330 mm long steel 45G2 and 325 mm long Granite block were taken. The results of the experiments are shown in Fig. 6 and Fig. 7.





Fig. 7. 325 mm long Granite block Temperature change over time.

The characteristics of the mentioned materials are: density of 45G2 ρ =7800 kg/m³, heat capacity Cp=500 J/(kg·K) and thermal conductivity coefficient λ =38.2 W/(m·K), and density of Granite ρ =2700 kg/m³, heat capacity Cp=790 J/(kg·K) and thermal conductivity coefficient λ =2.4 W/(m·K).

It took 700 s for the heat flow to reach the isolated surface of steel 45G2, and 5200 s for granite. Theoretical calculations using equation (1) give the following data: 688 s for steel and 5300 s for Granite.

From the comparison of theoretical and experimental results, it can be seen that the error of thermal conductivity coefficients measured by the presented method does not exceed 1.7% for steel and 1.9% for Granite.

CONCLUSION.

The creation of an energy audit device, which allows the accurate determination of the thermal conductivity (λ) coefficient and energy losses, ensures the performance of appropriate measurements to assess the thermal characteristics of buildings, energy losses and the level of energy efficiency of solid heating furnaces. This device can be used for both purposes as on-site and for the real-time measurements and recording the relevant data, as well as for the educational traning in the teaching laboratory.

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