INVESTIGATING THE STABILITY OF LONG ONE-DIMENSIONAL WAVES IN A SLOPING RUNOFF

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ABSTRACT

The paper is dedicated to the problem of the influence of waves in shallow-water slope flows on the intensity of soil erosion that has not been considered earlier. The stability of one-dimensional continuous waves on the free surface of the sloping runoff is analyzed, both at constant, and with the variable flow along the way. Attention is drawn towards the unsteady flow process and the shape of the free surface in various planes.

Introduction. Waves on the slopes of landscapes transport changes of the hydraulic and hydrological parameters of the flow (decreasing or increasing the flow rate, velocity, depth) both continuously and stepwise. One-dimensional continuous waves belong to the first type of waves, dynamic waves to the second type. Continuous wave on slopes are formed during heavy rain, especially during precipitation with variable intensity. The waves that arise in this case are often characterized by considerable amplitude, which increases both the erosive and transport ability of solid particles in the flow. These circumstances are often not taken into account when assessing the eroding flow rate, which affects the rate of soil erosion [1, 2, 3].

The paper is dedicated to the problem of the influence of waves in shallow-water sloping flows on the intensity of soil erosion that has not been considered earlier.

The aim of this paper is to study the stability of one-dimensional continuous waves on the free surface sloping runoff, both at constant and variable flow rate along the path, taking into account the unsteady flow process and the shape of the free surface in various planes.

Research results. In the articles [4, 5], one-dimensional continuous wave velocity calculated under the condition of continuity of water as a variable and a constant flow rate along a path, passing through the control volume of water, and moving at a velocity V_w. The stability of waves on the free surface of sloping runoff is analyzed. At the uniform mode of velocity V, the velocity of a continuous wave V_w is one and a half times greater than the average flow velocity over a cross section. The minimum depth of the sloping runoff H, at which the occurrence of waves on a free flow surface is:

$$H > \sqrt[y]{2n\sqrt{\frac{g}{i}}},\qquad(1)$$

where: n - roughness factor of the slope;
i - slope of the flat surface;

$y = 1{,}5\sqrt{n}$ - Exponent, when $H < 1m$.

Continuous waves will have appropriate values of the depth and besides each wave will spread with its velocity in accordance with the equation:

$$V_w = \frac{dq}{dH},\qquad (2)$$

where: q - flow rate per unit of slope width.

If, at the initial moment of runoff formation $t = 0$ at $X = 0$, then from this moment the waves will start spread appropriate to all values of H. According to the equation (2), at further flow movement, waves with high values of H will be transported faster. After a certain period of time t, the wave will pass a distance:
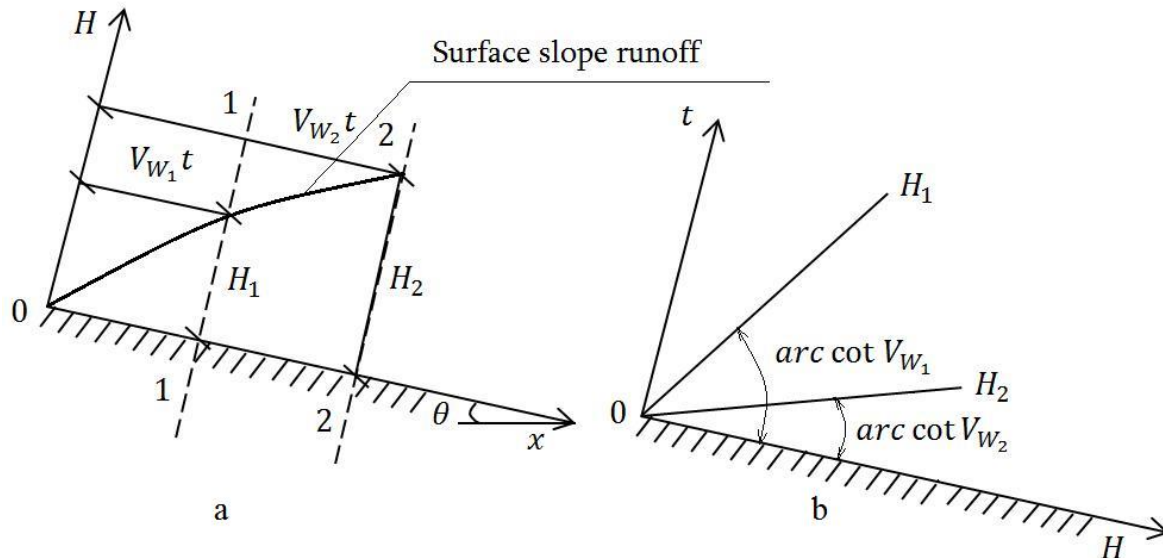
$$X = V_w t.\qquad (3)$$

Fig. 1. Sloping runoff - a) the profile of the runoff surface after the expiration of the time t; b) the position of the waves to the plane H0t.

When the flow runs down to the plane with a constant flow rate of depth H_1 and the flow rate instantaneously reducing towards a new value, which is corresponding to a stationary flow of depth H_2, then from point $x = 0$ will propagate waves, now they will correspond to the values H, enclosed within H_1 and H_2 (Figure 1). The initial stepwise disturbance will be distributed along the slope in the form shown on Figure 2.
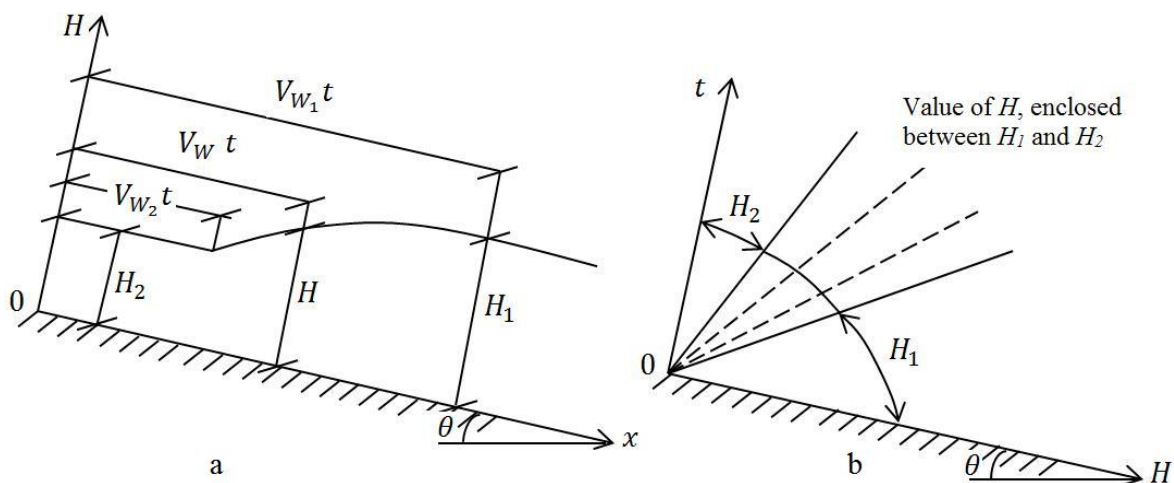

Fig. 2. Free surface of runoff after an instant decrease of flow rate - a) surface profile after the expiration of the time t; b) the position of the waves to the plane H0t.

The two previous examples show how the movement of continuous waves makes it possible to carry "final conditions" along the flow. This mechanism is common. If continuous waves are not

distributed in this way, then the final conditions can not have any effect. The boundary conditions are reached when the continuous waves are stopped, which in the case of elastic waves is analogous to locking, when the Mach number reaches unity.

Consider the case of flow movement with variable water flow along the path, then:

$$q_n \partial x + Q = (Q + \partial Q) + \frac{\partial \omega}{\partial t} dx, \tag{4}$$

where: q_n - increase of flow rate on a sloping section with length ∂x.

Transforming the equation (4), we obtain the equation of continuity for the flow with a variable flow rate along the way [3, 4, 5]:

$$\frac{\partial \omega}{\partial t} + V_w \frac{\partial \omega}{\partial x} = q_n . \tag{5}$$

It is not difficult to see that the left-hand side of equation (5) expresses the total time derivative of t from the ω plane for the coordinate system (in the one-dimensional treatment) moving with the velocity V_w in the direction of the OX axis.

For $q_n = 0$, we are dealing with a flow with a constant flow rate along the path, as discussed above.

Consider the following case:

$$\frac{dH}{dt} = q'_n = const. \tag{6}$$

Then for a plane flow we will obtain:

$$H - H_0 = q'_n (t - t_0), \tag{7}$$

where: *0* - index, which indicates the initial condition;

q'_n - flow rate of the connected flow per unit length and per unit width of the slope (q'_n – it has the dimension of velocity).

Taking into account that the wave velocity for a plane flow is expressed by the following equation:

$$V_w = \frac{dq_n}{dH}, \tag{8}$$

and for this case the equation can be written in the following form [1]:

$$V_w = \frac{dx}{dt} = 1{,}5C\sqrt{i}H^{0,5}, \tag{9}$$

where: *C* - Chezy coefficient.

Then, according to equations (6) and (9), we may obtain:

$$\frac{dH / dt}{dx / dt} = \frac{dH}{dx} = \frac{q'_n}{1{,}5C\sqrt{i}H^{0,5}} . \tag{10}$$

After integration of equation (10), taking into account the boundary conditions and simple transformations, we will obtain:

$$H = \sqrt[1,5]{H_0^{1,5} + \frac{q'_n (x - x_0)}{C\sqrt{i}}} . \tag{11}$$

Dependence (11) describes the trajectory of the waves surface on the plane $H0X$.

Movement instability in Newtonian liquids arises when the velocity of continuous one-dimensional waves V_w exceeds the velocity of dynamic waves C_1 propagating along the surface of the runoff, i.e.:

$$V_w > V + C_1 . \tag{12}$$

When the flow moves with a variable flow rate along the path, from equation (12) we will obtain the inequality [6, 7, 8, 9, 10]:

$$1{,}5 > 1 + \frac{\sqrt{g}}{C\sqrt{i}} . \tag{13}$$

Equation (13) indicates the loss of stability of the initial motion and arise of long continuous waves on the free surface of the sloping flow.

These equations can describe two types of waves: the first - when the waves are formed at the initial moment at $t_0 = 0$ from the initial alignment, i.e. for $x = 0$, then from (11) we will obtain:

$$H = \sqrt[1,5]{H_0^{1,5} + \frac{q'_n x}{C\sqrt{i}}} . \tag{14}$$

Eliminating H from (7) and (14), we obtain the equation of the wave distribution line in the $t0x$ plane as a function of the initial parameters:

$$[H_0 + q'_n(t - t_0)]^{1,5} = H_0^{1,5} + \frac{q'_n(x - x_0)}{1,5\ C\sqrt{i}}. \tag{15}$$

On the other hand, excluding the initial depth H_0 from (7) and (14), we define H in the function x at a given time, i.e. surface profile:

$$H^{1,5} = [H - q'_n(t - t_0)]^{1,5} + \frac{q'_n(x - x_0)}{1,5C\sqrt{i}}. \tag{16}$$

The conditions of this problem are satisfied by two types of waves: waves starting at the instant $t_0 = 0$ from the point $x_0 = 0$ for all values of H and waves starting at the subsequent moments from the point $x = 0$ at the value $H_0 = 0$.

For the second type of waves, from (7) - (16) we have:

$$H^{1,5} = H_0^{1,5} + \frac{q'_n(x - x_0)}{1,5C\sqrt{i}}. \tag{17}$$

The wave distribution lines to the $x0t$ plane are described by the following equation:

$$(H_0 + q'_n t)^{1,5} = H_0^{1,5} + \frac{q'_n x}{1,5C\sqrt{i}}, \tag{18}$$

and the surface profile by the equation:

$$H^{1,5} = (H - q'_n t)^{1,5} + \frac{q'_n x}{1,5C\sqrt{i}}. \tag{19}$$

For the second type of waves, the distribution lines and the surface profile coincide and are described by a single equation:

$$H^{1,5} = \frac{q'_n x}{1,5C\sqrt{i}}, \tag{20}$$

which is the equation of the runoff profile in the final steady state.

In the plane $H0t$, the distribution lines are parallel to each other (Figure 3) and from equation (15) it follows:

$$t = t_0 + \frac{\sqrt[1,5]{\dfrac{q'_n x}{1,5C\sqrt{i}}}}{q'_n}. \tag{21}$$
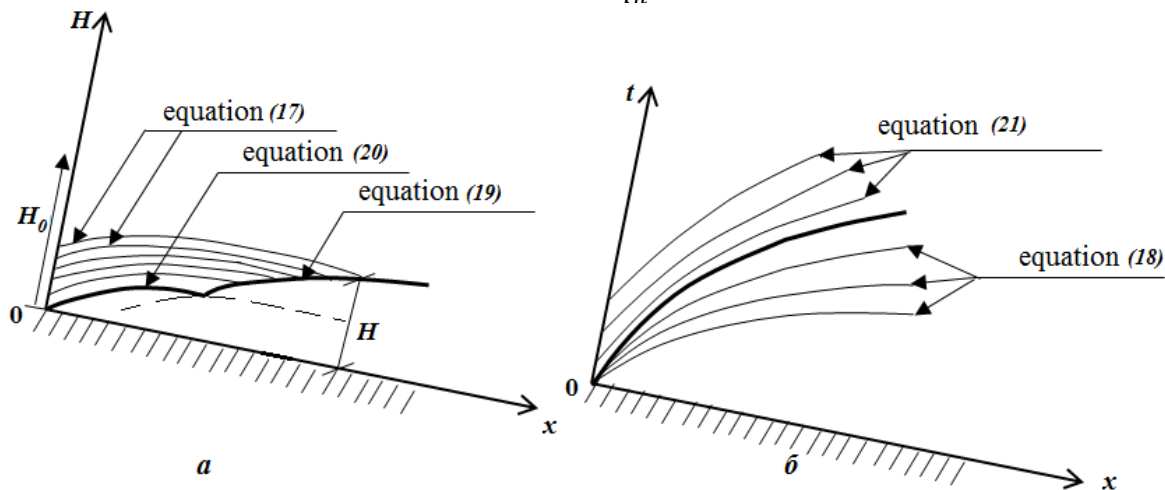

Fig. 3. Unsteady flow process at q_n (uniform inflow): a) surface profile of the runoff and wave distribution line; b) wave distribution lines.

Conclusions. According to the obtained equations, it is possible to predict the appearance of a wave on the free surface of the sloping runoff (both at constant and variable flow along the path), to estimate the magnitude of the wave velocity and the shape of the free flow surface in different planes ($H0X, t0X$).

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