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MATHEMATICAL MODEL OF TRAFFIC FLOW DISTRIBUTION ACROSS THE ROADS NETWORK

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ABSTRACT

Due to the military conflict in Ukraine, there is a problem of restoring the road network through partial or complete reconstruction in accordance with the traffic flow intensities. Therefore, it is important to conduct an analysis of the functioning state of the road network. In areas with complex traffic conditions, there is a need for traffic management based on a model of traffic flow distribution using mathematical modeling methods.

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Research Objective.

The aim of this research is to develop a mathematical model for the distribution of traffic flows.

Problem Statement.

The war in Ukraine has had a significant impact on the country's traffic flow. Roads, bridges, and other infrastructure have been damaged or destroyed as a result of the conflict, making it difficult for people to travel. Additionally, many individuals have been displaced due to the hostilities, leading to increased traffic congestion on roads and public transportation. Furthermore, checkpoints have been established throughout the country, complicating safe movement for people.

In the future, an increase in traffic intensity on the roads is expected in Ukraine. Improving road conditions is linked to increased attention from the government towards the development of the country's infrastructure, particularly roads. The government has announced plans to invest over 1$ billion in improving the country's transportation infrastructure over the next three years. It is expected that these investments will lead to better road conditions and increased traffic volumes. Additionally, the government has also announced plans to introduce new toll roads, further increasing traffic intensity. This calls for economic and mathematical justification, specifically in the form of models for predicting the distribution of traffic flows based on various factors such as route length, travel speed, traffic intensity, network condition, and other variables.

The distribution of traffic flows on a network segment with reduced capacity will help reduce congestion, which has a positive impact on travel speed and safety levels.

Analysis of Recent Research and Publications.

The mathematical model for traffic flow distribution developed by Michael J. Cassidy [1] is based on the concept of a "network of traffic flows," where each node represents a point where vehicles enter or exit the system, such as intersections or road entrances. Each link connects two nodes in the network. The model utilizes mathematical equations to calculate the expected traffic volume at each node, taking into account certain input data such as road conditions, velocity values, and other factors. The model considers the influence of driver behavior according to the traffic flow scheme.

The mathematical model for traffic flow distribution developed by Wilfrid S. Dixon [2] is based on the idea that traffic flows can be modeled as a stochastic process. The model assumes that the traffic flow is a random process influenced by both external and internal factors, including road conditions, congestion formation, and time-dependent effects. Additionally, it takes into account the impact of external factors such as weather, holiday periods, and special events. The model predicts the quantity of transportation moving between two points within a specific time period. Its objective is to forecast traffic volumes between two points at any given moment. The model is based on a set of equations that describe the influence of various parameters on traffic volumes over time. These equations consider factors such as road conditions, time of day, and congestion formation. The equations are used to estimate the number of vehicles for a given time period. Dixon's model has been utilized in the development of efficient public transportation routing systems and road network planning.

The mathematical model for traffic flow distribution developed by Scott S. Logan and John D. Powell [3] utilizes a combination of linear and dynamic programming to optimize the volumes of traffic for vehicles on a road network. The model takes into account factors such as the number of lanes on each road, speed limitations, and traffic intensity on each road segment in order to minimize congestion and increase the capacity of the network.

Mathematical model for traffic flow distribution developed by Stephen J. Yurkovich [4] utilizes graph theory and employs linear programming methods with consideration of constraints.

In the study by Hanyi Yang, Lili Du, Guohui Zhang, Tianwei Ma [5], developed a traffic flow dependency and dynamics based deep learning aided approach (TD^2-DL), which predict network-wide high resolution traffic speed propagation by explicitly integrating temporal-spatial flow dependency, traffic flow dynamics with deep learning method techniques. The authors first develop a graph theory-based method to identify the local temporal-spatial traffic dependency of each road among neighboring roads adaptive to the prediction horizon and traffic delay.

Presentation of the main research material.

The developed model for transportation flow distribution, considering the influence of obstacles, is based on the optimality conditions of flow distribution that involves bypassing obstacles. This model differs from existing ones in that, in addition to minimizing the total travel time of vehicles through a network of roads, it prevents congestion at network links and intersections, avoids vehicle retracing on previously traveled road segments, and incorporates local traffic flows by adjusting their values as each obstacle is introduced.

To solve the distribution of transportation flows through a network of roads, it is necessary to solve a system of equations (1):

$$\sum_{z=1}^{m_n} a_{k,z} \left(l_z \frac{V_z - \frac{\partial V_z}{\partial N_t(k)} \left(N_{M,Z} + \sum_{k=1}^{M} a_{k,z} N_T(k)\right)}{V_z^2} + \sum_{q=1}^{m_n} b_{z,q} a_{k,q} \frac{\partial t_{z,q}}{\partial N_t(k)} \right) + \lambda_{\xi,\eta} = 0 \tag{1}$$

$$\sum_{k(\xi,\eta-1,\ 1)}^{k(\xi,\eta-1,\ n_{\xi,\eta})} N_T(k) - N_{\xi,\eta} = 0.$$

$a_{k,z}$ - matrix element equal to 1 if there is a match and 0 if there is no match; $N_{M,Z}$- traffic intensity of the unallocated traffic flow along the *z*-th line; l_z - length of the z-th line; V_z - average speed of traffic on the *z*-th section, which depends on the traffic intensity on it; $b_{z,q}$ - matrix element indicating the existence of a connection between the runs, the connection exists when $b_{z,q}$ = 1, there is no connection when, z≠q $b_{z,q}$ = 0, *z*,*q*=1,2,…m(n), z≠q; *N(k)* - traffic intensity of distributed traffic flows along *k* traffic routes; $N_T(k)$ - distributed traffic flow along the *k*-th route; $N_{\xi,\eta}$ - incoming traffic flow; *z* – number of the race.

The specified system is presented in a general form, for the solution it is necessary to specify the dependencies.

Dependencies can be taken in the form of:

$$v=a-b\,N,$$

$$t=d+c\,N. \tag{2}$$

The use of linear dependencies is justified not only by the simplicity of their derivation under different road conditions but also by their sufficient reliability in describing the actual process of traffic flow on a network of roads and the formation of delays for vehicles at intersections. This is supported by numerous studies.

The utilization of linear dependencies is justified not only due to their straightforward derivation in various road conditions but also due to their sufficient reliability in describing the actual process of traffic flow on a network of roads and the formation of delays for vehicles at intersections. This claim is substantiated by numerous research studies. (6, 7, 8 ,9)

The objective function will take the following form (3):

$$T=\sum_{z=1}^{m_n}\left(\frac{l_z\,(N_{M,Z}+\sum_{k=1}^{M}a_{k,z}N_T(k))}{a_z-b_z(N_{M,Z}+\sum_{k=1}^{M}a_{k,z}N_T(k))}+\sum_{q=1}^{m_n}\right.$$
$$\left.d_{z,q}+c_{z,q}\left(N_{M,Z,q}+b_{z,q}\sum_{k=1}^{M}a_{k,z}a_{k,q}N_T(k)\right)\right); \tag{3}$$

a_z - free speed on the line; b_z - correlation coefficient; $d_{z,q}$ - time of the intersection passing; $c_{z,q}$ - average vehicle delay time before the intersection.

In this objective function, the dependencies *v=f(N)* and *t=φ(N)* have ranges of *N* values from minus infinity to infinity:

$$t=\frac{l_z\,(N_{M,Z}+\sum_{k=1}^{M}a_{k,z}N_T(k))}{a_z-b_z(N_{M,Z}+\sum_{k=1}^{M}a_{k,z}N_T(k))} \tag{4}$$

the time of movement of all cars in the *z*-th race in the case of accounting for the local intensity of traffic in the race, which can be written in general form:

$$t_{n,м}=\frac{l_z\,(N_{n,м}+N_{n,m})}{a_n-b_n\left(N_{n,м}+N_{n,m}\right)}\;; \tag{5}$$

without taking into account the local traffic intensity:

$$t_n = \frac{l_n N_n}{a_n\text{-}b_n \ \ N_{n,м}} \ , \qquad (6)$$

$N_{п,м}$, $N_{п,т}$ - according to the intensity of local and transit traffic flows on the route; $a_п$ - free average speed in the race; $b_п$ - correlation coefficient per race.

Expressions (4), (6) indicated in a general form have a graphic image presented in Figure 1.

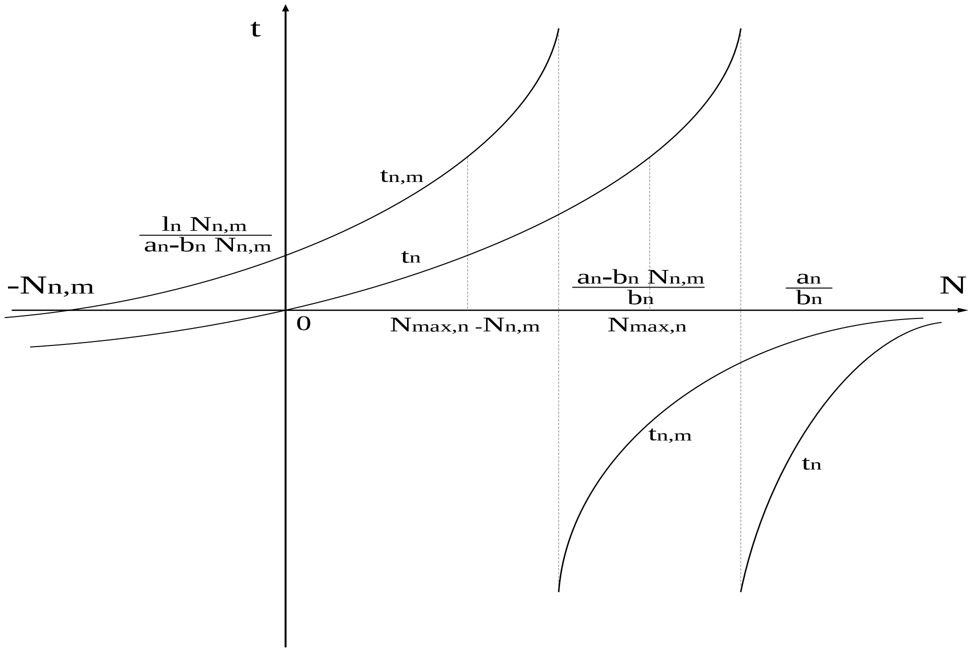

Figure 1. Graph of the function $t_{n,m}, t_n$

$$t_{y,z,d}=d_{z,q}+ c_{z,q}(N_{M,\ Z,q}+b_{z,q}\sum_{k=1}^{M} a_{k,z} \ \ a_{k,q} \ N_T(k) \ )$$

The delay time for all vehicles at the intersection between the z-th and q-th roadway segments refers to the amount of time that vehicles are delayed while passing through the intersection. It represents the additional time it takes for vehicles to travel from the z-th segment to the q-th segment due to congestion and other factors at the intersection.

If the local traffic intensity at the intersection in a given direction is taken into account or not, the formula will have the following general form:

$$t_{y,M}=d_y+c_y\left(N_{y,м}+N_{y,m}\right)$$

$$t_y=d_y+c_y \ N_y \qquad (7)$$

Expressions have a graphic appearance Figure 2,

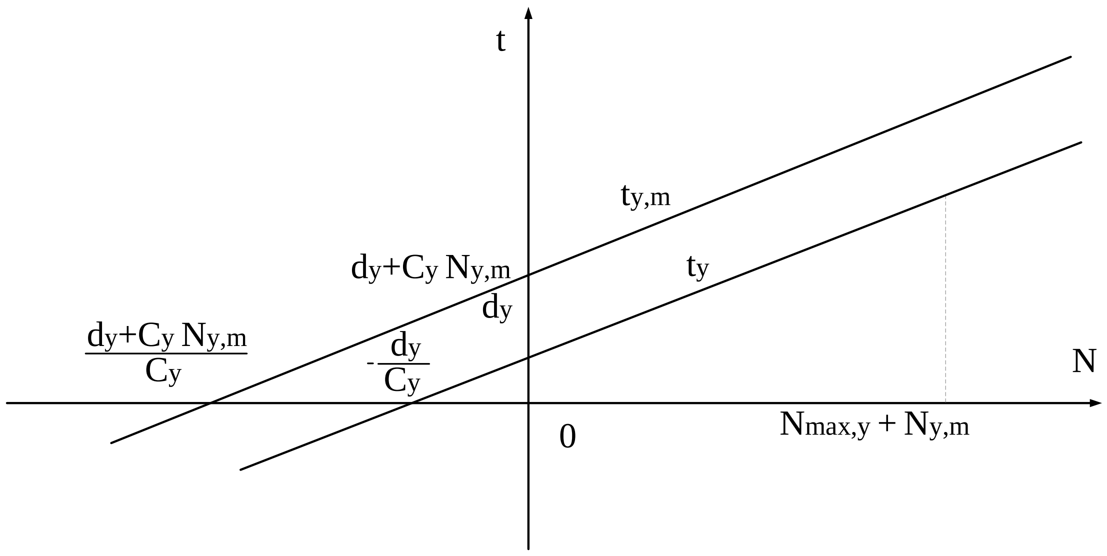

Figure 2. Graph of the function $t_{y,m}$, t_y

where $t_{y,M}$ – delay at the node taking into account local $N_{у,м}$ and transit $N_{у,т}$ traffic flows; d_y – parameter; c_y – parameter; N_y – traffic flow on the road network; t_y – node delay of all distributed flow.

To analyze the state of traffic flows on road sections under the influence of traffic obstacles in a given section, it is necessary to build and analyze mathematical models of the optimal distribution of traffic flow on the road before and after the occurrence of an obstacle.

A necessary and sufficient condition for the optimal distribution of traffic flows is the equality of the first partial derivatives of the total travel time on sections and intersections of routes of a given direction.

Consider the first parts of the derivatives of functions $t_{п,м}$, $t_п$, $t_{y,M}$, t_y;

$$\frac{\partial t_{П,М}}{\partial N_{П,Т}} = \frac{l_П a_П}{(a_n - b_n\left(N_{n,м}+N_{n,m}\right))^2},$$

$$\frac{\partial t_П}{\partial N_П} = \frac{l_П a_П}{(a_n - b_n N_n)^2},$$

$$\frac{\partial t_{y,м}}{\partial N_{y,m}} = c_y,$$

$$\frac{\partial t_y}{\partial N_y} = c_y.$$

Let's present them graphically in Figure 3.

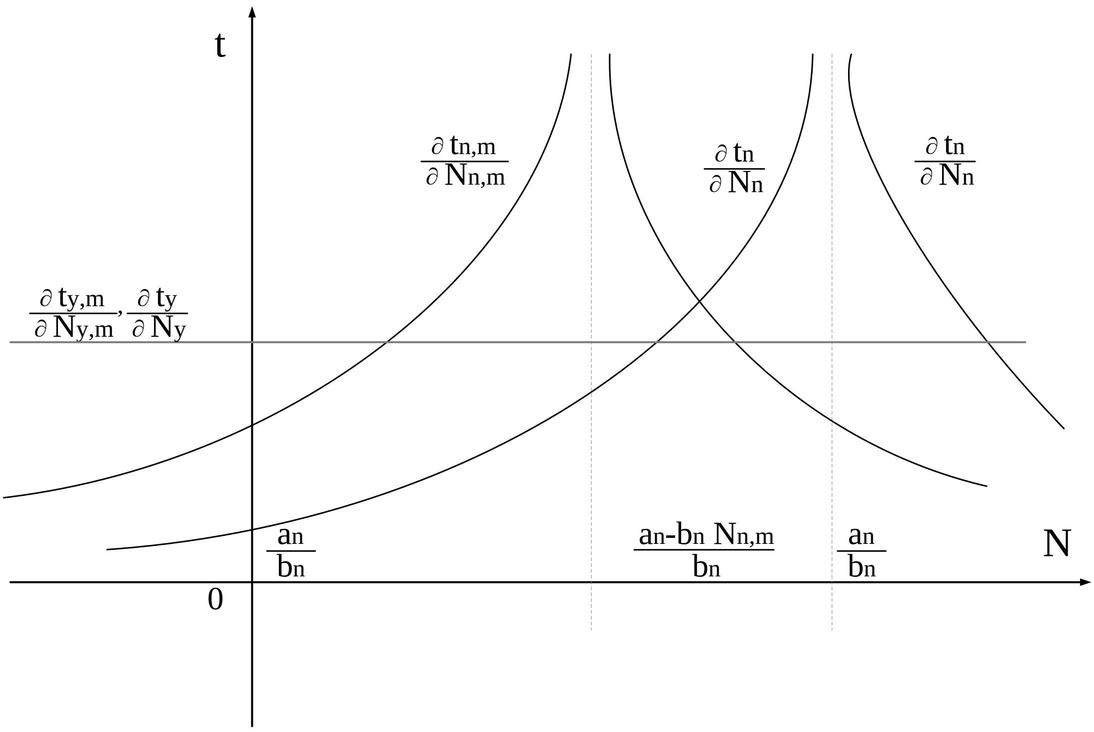

Figure 3. Graph of the first derivatives of the function $t_{n,m}$, t_n, $t_{y,m}$, t_y

The graph shows that the first part of the derivative $\partial t_{П,М}/\,\partial N_{П,Т}$ within the definition from minus infinity to $(a_{п} - b_{п}\, N_{п,м})/\, b_{п}$ is a monotonically increasing continuous function, within the limits of $(a_{п} - b_{п}\, N_{п,м})/\, b_{п}$ to infinity, is a monotonically decreasing continuous function, at $N_{П,Т} = (a_{п} - b_{п}\, N_{п,м})/\, b_{п}$ the value of the derivative tends to infinity.

The first part of the derivative $\partial t_{П}/\,\partial N_{П}$ is a function defined in the range from minus infinity to $a_{п}/b_{п}$ and is a monotonically increasing function, defined in the range from $a_{п}/b_{п}$ to infinity and is a continuous monotonically decreasing function, with $N_{П} = a_{п}/b_{п}$ the value of the derivative tends to infinity.

The first parts of the derivatives $\partial t_{у,м}/\partial N_{у,т}$, $\partial t_{у}/\partial N_{у}$ are continuous functions defined in the region from minus infinity to plus infinity, the values of which remain constant.

At *N = 0*, the functions $t_{п,м}$, $t_{п}$, $t_{y,M}$, t_{y} have different values due to different traffic conditions both on the runs and at intersections. Since the derivatives for each route of the given directions are the sum of the derivatives for the races and intersections, they are also continuous and have different values at *N=0*. Figure 4.

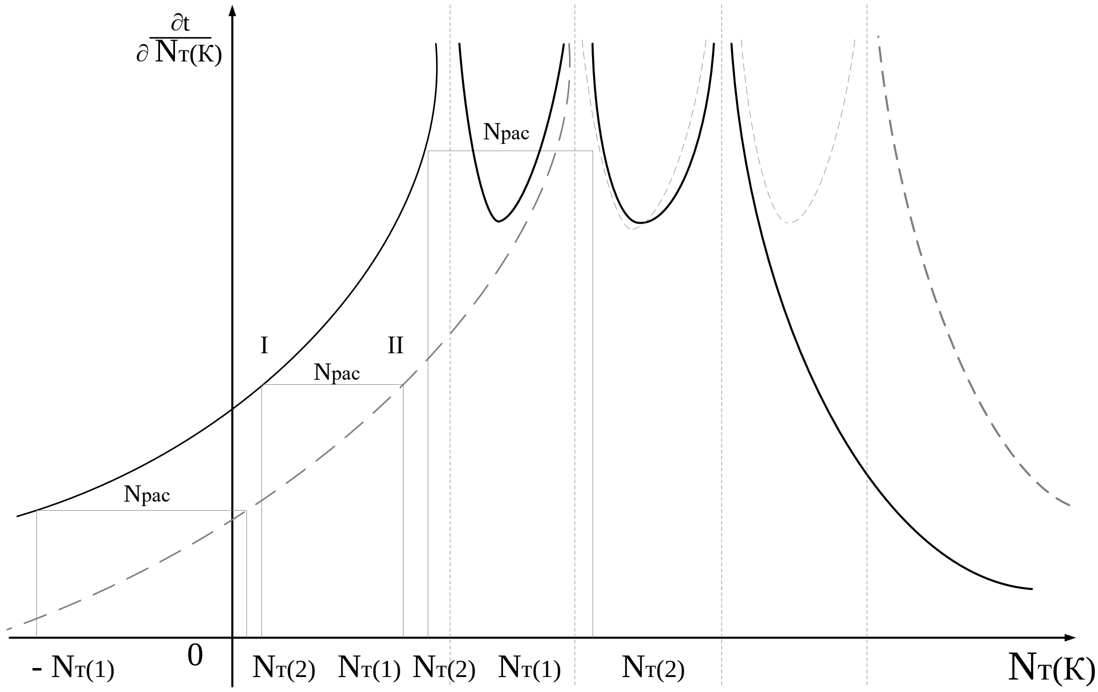

Figure 4. Graph of first derivatives on routes I and II

Figure 4 shows that the conditions for equality of derivatives in the presence of a distributed traffic flow are always met. In this case, condition (7) will be met, since it will always be possible to find such an algebraic sum of $N_T(1)$ and $N_T(2)$ that is equal to the flow distributed in the directions. An algebraic sum can be defined as the sum of numbers or their difference. Figure 4 shows possible cases of equality of derivatives when condition (7) is met.

Situation 1: Condition (7) is written:

$$N_T(2) - N_T(1) = N_{pac} .$$

Situation 2. Condition (7) will be written:

$$N_T(1) + N_T(2) = N_{pac} .$$

Situation 3. Condition (7) will be written:

$$N_T(1) + N_T(2) > N_{pac} .$$

Thus, in the case of the first route, the second route has such a capacity reserve that it can take on an additional amount of local traffic from the first route equal to $N_T(1)$. This solution makes mathematical sense, but not physical sense, since it is always possible to redirect the local traffic flow of the first route along the second route.

In order to ensure that the mathematical model of traffic flow distribution corresponds as closely as possible to the physical nature of the traffic process on the network, two approaches should be considered.

The first implies restrictions on the size of distributed flows on the part of the capacity of the lanes and intersections.

Based on the physical essence of the process, it is necessary to construct the functions $v=f(N)$ *i* $v=\varphi(N)$ in such a way that:

$$t_{П,М} \rightarrow \infty \text{ at } N_{max,П} - N_{П,М} < N_{П,Т} < N_{max,П} - N_{П,М} + \delta;$$

$$t_{П} \to \infty \ at \ N_{max,П} < N_{П} < N_{max,П} + \delta;$$

$$t_{П,M} \to \infty \ at \ N_{max,П} - N_{y,M} < N_{y,T} < N_{max,y} - N_{y,M} + \delta;$$

$$t_{y} \to \infty \ at \ N_{max,y} < N_{y} < N_{max,y} + \delta; \quad (8)$$

where $\delta \to \infty$, $\delta \neq \infty$

At $0 \leq N_{П,М} \leq N_{max,П} - N_{П,М}$; $0 \leq N_{П} \leq N_{max,П}$; $0 \leq N_{y,T} \leq N_{max,y} - N_{y,M}$; $0 \leq N_{y} \leq N_{max,y}$, the functions $t_{П,М}, t_{П}, t_{y,M}, t_{y}$ are not subject to change, they reflect the real movement. Consequently, the derivatives of the traffic routes at $N = 0$ will have different values other than 0, and at $N = N_{max} + \delta$ they tend to infinity, which leads to the exclusion of case 3 with the possible occurrence of case 1.

Therefore, to solve the problem of optimal distribution of traffic flows on the street and road network when bypassing the zone of influence of a traffic obstacle, it is necessary to exclude from the entire set of routes those routes for which the values of the distributed traffic flows become negative.

Such a solution is only possible if the distribution of traffic flows is based on priority traffic directions. In the case when priority directions cannot be assigned, such a solution leads to significant time spent on obtaining it. Therefore, the second approach to solving the problem is considered, which, while maintaining the restrictions of the first approach, in turn, excludes the possibility of obtaining negative values of distributed flows.

Based on the physical essence of the process, it is necessary to construct the functions $v = f(N)$ i $v = \varphi(N)$ in such a way that:

$t_{П,M} \to \infty \ at \ N_{max,П} - N_{П,M} < N_{П,T} < N_{max,П} - N_{П,M} + \delta; \quad N_{П,T} < 0;$
$t_{П} \to \infty \ at \ N_{max,П} < N_{П} < N_{max,П} + \delta; \quad N_{П} < 0;$
$t_{П,M} \to \infty \ at \ N_{max,П} - N_{y,M} < N_{y,T} < N_{max,y} - N_{y,M} + \delta; \quad N_{y,T} < 0;$
$t_{y} \to \infty \ at \ N_{max,y} < N_{y} < N_{max,y} + \delta; \quad N_{y} < 0;$

$$\frac{\partial t_{П,M}}{\partial N_{П,T}} = 0 \quad at \quad N_{П,T} = 0$$

$$\frac{\partial t_{П}}{\partial N_{П}} = 0 \quad at \quad N_{П,T} = 0$$

$$\frac{\partial t_{y,M}}{\partial N_{y,T}} = 0 \quad at \quad N_{y,T} = 0 \quad (9)$$

$$\frac{\partial t_{y}}{\partial N_{y}} = 0 \quad at \quad N_{y} = 0,$$

where $\delta \to \infty$, $\delta \neq \infty$.

The fulfillment of conditions (9) leads to the fulfillment of the constraints according to which the minimum of the criterion will always be in the region of positive values. This gives a certain decision error, but its value is manageable, but not equal to zero.

Conclusions.

As a result, it was proved that mathematical modeling can be used to analyze various options for developing traffic management measures, including solving the problem of congestion on a particular section of the road.

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