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# DECISION MAKING UNDER LINGUISTIC UNCERTAINTY CONDITIONS ON BASE OF GENERALIZED FUZZY NUMBERS

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## ABSTRACT

This article is devoted to the problem of decision making under linguistic uncertainty. The effective method for modelling linguistic uncertainty is the fuzzy set theory. There are several types of fuzzy number types proposed by L. Zadeh: fuzzy type-1, fuzzy type-2, Z-numbers. Chen proposed concept of generalized fuzzy numbers. Generalized trapezoidal fuzzy numbers (GTFN) one of effective approach which can be used for modeling linguistic uncertainty. GTFN very convenient model which allow take in account second order uncertainty. GTFN are formalized and major operations are described as practical problem is considered group decision making for supplier selection. In this case the criteria assessments are expressed by experts in linguistic form. Group decision making model is presented as 2 step aggregation procedure, in first step is aggregated value of alternative by expert, in second step by criteria. Numerical example with four criteria and three alternatives are presented and solved.

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1. **Introduction.** Decision making problem with imperfect information is very actual problem. As known in many practical cases we need to be satisfied of expert information and the linguistic assessments. One is effective method of modelling linguistic information is fuzzy set approach. There are many scientific works dedicated to applications of classical fuzzy approach which is named fuzzy type-1 proposed by L. Zadeh (1965) [1]. In 1975 L. Zadeh [2] proposed more general approach fuzzy type-2, which expands the features of classical fuzzy type-1 model. Chen in 1985 proposed generalized fuzzy set concept [3]. L. Zadeh in 2011 proposed fuzzy Z-numbers approach [4]. All these approaches allow not only modelling our imprecise knowledge about factors and also take in account our imprecision about membership function. All these models have more powerful features for modelling uncertainty [6-16].

2. **Preliminaries.** In this article we discuss about application of generalized trapezoidal fuzzy numbers (GTFN) for modelling MADM problem [6].

**Definition: General fuzzy number.** A fuzzy set  $\tilde{A}$ , defined on the universal set of the real numbers  $R$ , is said to be generalized fuzzy number if its membership function has the following characteristics:

- (i)  $\mu_{\tilde{A}}: R \rightarrow [0,1]$  is continuous
- (ii)  $\mu_{\tilde{A}}(x) = 0$  for all  $x \in (-\infty, a] \cup [d, \infty)$
- (iii)  $\mu_{\tilde{A}}(x)$  is strictly increasing on  $[a, b]$  and strictly decreasing on  $[c, d]$
- (iv)  $\mu_{\tilde{A}}(x) = w$ , for all  $x \in [b, c]$ , where  $0 < w \leq 1$ .

Generalized trapezoidal fuzzy number  $\tilde{A} = (a, b, c, d, w)$  is said to be generalized fuzzy number if its membership function is given

$$\mu_x(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ w & b \leq x \leq c \\ \frac{x-c}{d-c} & c \leq x \leq d \\ 0 & x > d \end{cases}$$

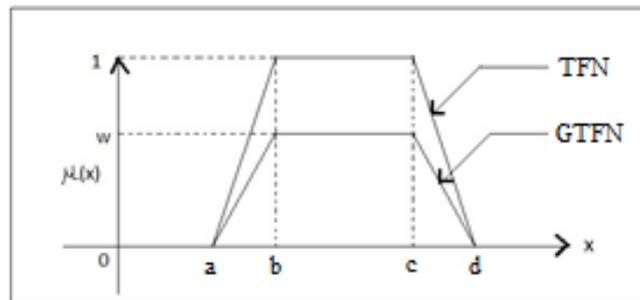


Fig 1. Comparison between membership function of TFN and GTFN

Here  $W$  plays role of confidence level.

Consider arithmetical operations on two trapezoidal GTFN numbers:  $\tilde{A}_1$  and  $\tilde{A}_2$  numbers are given:

$$\tilde{A}_1 = (a_1, b_1, c_1, d_1, w_1) \quad \tilde{A}_2 = (a_2, b_2, c_2, d_2, w_2)$$

**Addition**

$$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2))$$

**Subtraction**

$$\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2; \min(w_1, w_2))$$

**Scalar Multiplication**

$$\lambda A = \begin{cases} (\lambda a, \lambda b, \lambda c, \lambda d; w) & \lambda > 0 \\ (\lambda d, \lambda c, \lambda b, \lambda a; w) & \lambda < 0 \end{cases}$$

**Ranking function**

For ranking alternatives we have used following centroid method /6/

$$(\tilde{x}_0, \tilde{y}_0) = \left( a + b + c + d - \frac{dc - ab}{(dc) - (a + b)}, \frac{w}{3} \left( 1 + \frac{c - b}{(d + c) - (a + b)} \right) \right)$$

Ranking function

$$R(\tilde{A}) = \sqrt{\tilde{x}^2 + \tilde{y}^2} \tag{1}$$

Let  $\tilde{A}_i$  and  $\tilde{A}_j$  two fuzzy numbers,

- (i)  $R(\tilde{A}_i) > R(\tilde{A}_j)$  then  $\tilde{A}_i > \tilde{A}_j$
- (ii)  $R(\tilde{A}_i) < R(\tilde{A}_j)$  then  $\tilde{A}_i < \tilde{A}_j$
- (iii)  $R(\tilde{A}_i) = R(\tilde{A}_j)$  then  $\tilde{A}_i = \tilde{A}_j$

With GTFN we can represent the crisp interval and also imprecise interval. If  $a=b$  and  $c=d$  and  $W \neq 1$  we have imprecise interval with confidence level  $W$ .

If  $a=b, c=d$  and  $w=1$  then we have crisp interval.

**3. Problem statement and solving method**

Let's consider supplier selection problem with GTFN. This problem is formalized as MADM problem. Exist 3 potential suppliers  $A_i$  ( $i = 1,2,3$ ) and their activity are described by 4 attributes:

$C_1$ - raw quality,  $C_2$  – risk factor,  $C_3$  –service level,  $C_4$  – company profile.

Let's say that for decision making group of 3 experts established  $E_k$  ( $k = 1,2,3$ ) and corresponding weight coefficients are determined

$$\lambda = (0.3 \ 0.45 \ 0.25)$$

For 4 attributes  $C_i$  ( $i = 1,2,3,4$ ) are determined weight coefficients  $\omega = (0.3 \ 0.15 \ 0.2 \ 0.35)$

In table 1 are presented linguistic terms which will be used for alternative evaluation “Very Low” (VL), “Low” (L), “Medium” (M), “High” (H), “Very High” (VH) (Fig.2)

Table 1. Linguistic terms for alternative evaluation

Linguistic term	GTFN values
Very Low (VL)	(0,0.1,0.2,0.3;0.6)
Low (L)	(0.1,0.3,0.45,0.7;0.7)
Medium	(0.4,0.5,0.7,0.8;0.8)
High (H)	(0.5,0.6,0.75,0.85;0.85)
Very High (VH)	(0.6,0.7,0.8,0.9;1)

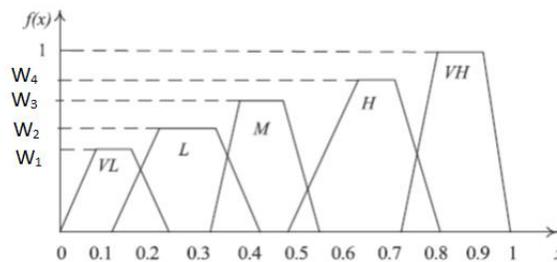


Fig. 2. Linguistic terms for alternatives evaluation

Experts using these terms have evaluated any potential suppliers and results are presented in following tables 3-5

Table 3 Alternatives evaluation by 1<sup>st</sup> expert

	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	M	H	VH	VH
$A_2$	H	M	H	H
$A_3$	VH	VH	M	H

Table 4 Alternatives evaluation by 2<sup>nd</sup> expert

	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	H	VH	H	H
$A_2$	M	H	VH	VH
$A_3$	H	VH	M	VH

Table 5. Alternatives evaluation by 3<sup>rd</sup> expert

	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	M	H	H	H
$A_2$	H	VH	VH	H
$A_3$	M	H	M	VH

First we carry out aggregation by experts using formula

$$\tilde{A}_{ij}^k = \oplus_{k=1}^3 (\lambda_k \tilde{A}_{ij}^{(k)})$$

and we have achieved following results:

- $\tilde{A}_{11}=(0.46,0.57,0.75,0.86;0,8)$
- $\tilde{A}_{12}=(0.55,0.65,0.77, 0.87;0.85)$
- $\tilde{A}_{13}=(0.53,0.63,0.77,0.87;0.8)$
- $\tilde{A}_{14}=(0.53,0.63,0.77,0.87;0.8)$
- $\tilde{A}_{21}=(0.46,0.56,0.73,0.83;0.8)$
- $\tilde{A}_{22}=(0.50,0.60,0.75,0.85;0.8)$
- $\tilde{A}_{23}=(0.57,0.67,0.79,0.89;0.85)$
- $\tilde{A}_{24}=(0.55,0.65,0.77,0.87;0.8)$
- $\tilde{A}_{31}=(0.73,0.79,0.87,0.92;0.8)$
- $\tilde{A}_{32}=(0.58,0.68,0.79,0.89;0.85)$
- $\tilde{A}_{33}=(0.40,0.50,0.70,0.80;0.8)$
- $\tilde{A}_{34}=(0.57,0.67,0.79,0.89;0.85)$

These results can be presented as collective decision matrix

$$R=\begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{A}_{13} & \tilde{A}_{14} \\ \tilde{A}_{21} & \tilde{A}_{22} & \tilde{A}_{23} & \tilde{A}_{24} \\ \tilde{A}_{31} & \tilde{A}_{32} & \tilde{A}_{33} & \tilde{A}_{34} \end{pmatrix}$$

On next step we carry out aggregation by attributes using formula

$$A_i = \bigoplus_{j=1}^4 (\omega_j \tilde{A}_{ij})$$

As result we have global evaluation of all alternatives (Table 6)

Table 6. Global evaluation of all alternatives

Alternatives	GTFN values
$A_1$	$(0.51,0.61,0.76,0.87;0,8)$
$A_2$	$(0.52,0.62,0.76,0.86;0.8)$
$A_3$	$(0.58,0.67,0.79,0.88;0,8)$

For comparison alternative decisions we will use Rank function (1)

$$\text{Rank}(A_1)=3.52 > \text{Rank}(A_3)=3.49 > \text{Rank}(A_2)=3.45$$

It means that best is supplier  $A_1$

**Conclusions.** In this article have been considered problem of MADM under linguistic uncertainty. As model of decision making used group decision making approach and as model for modeling uncertainty have been used GTFN model. As test problem for proposed model have been used the supplier selection problem.

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