International Journal of Innovative Technologies in Social Science

e-ISSN: 2544-9435

Scholarly Publisher
RS Global Sp. z O.O.
ISNI: 0000 0004 8495 2390

Dolna 17, Warsaw,
Poland 00-773
+48 226 0 227 03
editorial_office@rsglobal.pl

ARTICLE TITLE	A DIDACTIC SOLUTION FOR INTEGERS: A METHODOLOGICAL VARIANT WITH PRACTICAL DEMONSTRATION
ARTICLE INFO	Munkhtaria Khayankhyarvaa. (2025) A Didactic Solution for Integers: A Methodological Variant With Practical Demonstration. *International Journal of Innovative Technologies in Social Science*. 2(46). doi: 10.31435/ijitss.2(46).2025.3383
DOI	https://doi.org/10.31435/ijitss.2(46).2025.3383
RECEIVED	14 April 2025
ACCEPTED	12 June 2025
PUBLISHED	20 June 2025
LICENSE	The article is licensed under a **Creative Commons Attribution 4.0 International License.**

A DIDACTIC SOLUTION FOR INTEGERS: A METHODOLOGICAL VARIANT WITH PRACTICAL DEMONSTRATION

Munkhtaria Khayankhyarvaa
Ph.D., National Defense University, Mongolia

ABSTRACT

This study examines the effectiveness of using a symbol counter methodology to teach integer operations within a three-tiered didactic framework: concrete application, visual representation, and symbolic notation. The experimental approach was designed to address common student challenges, such as sign errors, incomplete understanding, and flawed reasoning in mathematical problem-solving. The pilot lesson incorporated student-centered teaching strategies, including hands-on activities, teamwork, discussion, and reflection.

The symbol counter, a tool utilizing physical symbols like paper circles with plus or minus signs, helped students grasp concepts such as positive and negative numbers, opposite numbers, and zero through tactile and visual experiences. This method facilitated a deeper understanding of integer properties and operations, including addition, subtraction, multiplication, and division. By transitioning from physical manipulation to mental visualization and symbolic abstraction, students were able to internalize mathematical concepts more effectively.

To validate the hypotheses proposed in the study, we explored the feasibility of employing methodological approaches that encourage student engagement in the lesson, critical thinking, self-directed learning, and collaboration. To achieve the established objectives, the following tasks were undertaken:

- Identifying the experimental group
- Creating the experimental lesson plan
- Conducting the experiment
- Evaluating the experimental outcomes

In high school math classes, we learned the rules for performing operations with integers. The sign calculations were written and formulated as follows:

1. (+) × (+) = (+)
2. (+) × (-) = (-)
3. (-) × (+) = (-)
4. (-) × (-) = (-)

These notations can be found in various textbooks and educational materials.

KEYWORDS

Symbol Counter, Didactic Solution, Addition, Subtraction, Multiplication, Division, Reflection, Integer

CITATION

Munkhtaria Khayankhyarvaa. (2025) A Didactic Solution for Integers: A Methodological Variant With Practical Demonstration. *International Journal of Innovative Technologies in Social Science*. 2(46). doi: 10.31435/ijitss.2(46).2025.3383

Introduction.

Students made numerous errors in marking, displayed flawed reasoning when solving problems, and exhibited various shortcomings, including incomplete understanding. Nonetheless, when employing a marker counter for teaching, both the benefits and drawbacks were noted, such as error-free marking, enhanced comprehension, and reduced incorrect reasoning. Various teaching approaches, including interviews, project-based learning, discussions, reflection, teamwork, case studies, and hands-on learning, were utilized in structuring the pilot lesson. The experiment conducted based on the refined methodology suggests that teaching with the marker counter, as outlined, can effectively facilitate instruction.

This lesson was structured around the fundamental concepts covered in previous classes and the knowledge and skills that students have acquired. The performance of tasks, individual work, and teamwork were assessed for each student during the testing of the alternative methodology. Problems were assessed and evaluated using a scoring system for each topic group. These topics include:

- Create pairs with a sum of zero
- Consider mathematical problems
- Calculate the difference
- Calculate the sum
- Calculate the product
- Calculate the quotient
- Apply logical thinking

Teaching methods for integer operations.

For someone who has a deep understanding of natural numbers and their operations, learning, studying, and teaching integers and their operations should not pose a significant challenge. It is important to grasp the concepts of negative numbers, opposite numbers, and zero, which are key distinctions between integers and natural numbers, as well as their properties.

The teaching approach for integers involves a series of activities that cover positive numbers, negative numbers, opposite numbers, zero, and their properties across three levels: practical application, visual representation, and symbolic notation.

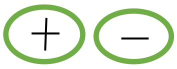
Therefore, the pedagogy of integers encompasses the teaching of zero as the sum of natural numbers, negative numbers, opposite numbers, and their properties in a cohesive manner.

At a practical level, positive, negative, opposite numbers, zero, and their mathematical meanings are translated into practical actions, demonstrating their mathematical significance. To illustrate this, teachers can utilize a counter, a tool that effectively represents integers and their operational properties. Any object that can be used with addition and subtraction signs can serve as a counter.

For instance, a paper circle or square with a plus or minus sign can be used as a counter by placing it inside the shape. In this way, we will provide a brief explanation and example of how integers and their operational properties can be illustrated using a counter.

A circle with a plus sign symbolizes the natural number one.

A circle with a minus sign symbolizes the number that is the opposite of one.



This set of counters is known as the zero pair.

Example 1. To illustrate adding one to one using counters: Place a circle on a table. Add another circle next to it ⊕ ⊕ on the table and count the total number of circles in this arrangement.

Example 2. To demonstrate adding the opposite of one to the opposite of one, you can use a counting symbol. ⊖ Place one figure on the table and another figure 2 on the table. Group them together side by side and count the total number of round figures in the group. You will see that two round figures with a minus sign equal 0.

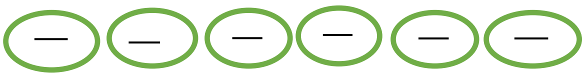
Example 3. To demonstrate adding one to the opposite of one using a sign counter, follow these steps: ⊖ Place one figure on a table. Then, place another figure on the table next to the first one to create a group. Count the total number of figures ⊖ ⊖ in the group and notice that their signs are opposite. Explain that this pairing results in a zero pair.

Example 4. To demonstrate adding the opposite of one to two, use a counter in the following way: ⊖ Place one figure on a table. ⊖ Add two more figures to the table, forming a group with the first one. Count the total number of figures ⊖ ⊖ ⊖ in the group and check for any zero pairs. Remove any zero pairs and see if there is one remaining figure with a minus sign in the group.

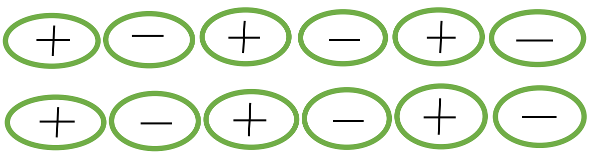
Example 5. Demonstrating the addition of two to the opposite of one using a counter can be done by taking a picture (e.g., placing it on a table) and then taking two more pictures (also placing them on the table). Group the three pictures together, count the total number of pictures in the group, and check if a zero pair can be identified. Once the zero pair is identified, check if one round picture with a plus sign remains. These include:

Negative two multiplied by negative three equals six. Adding zero to the product: six plus zero equals six. Subtracting the product from zero: zero minus six equals negative six. Expressing the product as a signed numerator: negative six.

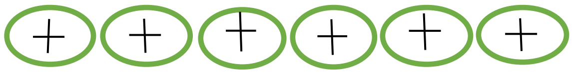
[(-2) * (-3) = 6, Adding zero to the product: 6 + 0 = 6, Subtracting the product from zero: 0 - 6 = -6, Expressing the product as a signed numerator: -6]



To subtract 6 zero pairs with a minus sign, you would need to have 6 minus signs. To demonstrate this operation practically, you can add 6 zero pairs.



By subtracting six wheels with a minus sign from the set created by adding pairs of zero's, it is evident that six wheels with a plus sign remain in the set after the removal.



The concept of integers can be effectively taught using a symbolic counter, as demonstrated in the addition of positive, negative, and opposite numbers.

The child's understanding of positive, negative, and opposite numbers, as well as zero, is a direct result of the practical work done at this level.

Through visualization, the tactile experience of positive, negative, and opposite numbers and zero, along with their mathematical significance, is translated into mental imagery and represented in drawings.

The teacher assesses and interprets the child's visualizations and drawings, providing guidance and feedback when necessary.

Hence, the teacher instructs the child to draw a number line, indicate the starting point, and utilize the integer number line to depict the integer. This process involves verbalizing the number associated with the position reached when adding a positive (negative) number to an integer. The child counts from the initial number on the number line in the appropriate direction (right or left) based on the number being added.

Some relevant examples are provided below. The didactic result of imagining and describing at the level of representation is evident when a child has experienced the properties of integers and their actions, and has depicted them on a chosen figure and number line.

At the symbolic level, the concepts of positive, negative, and zero numbers, along with their mathematical operations, are represented through mathematical language codes. The teacher should guide students to visually and mentally engage with these codes, fostering understanding and proficiency in integer operations. For instance, the equation $0 = a + (-a)$ illustrates that the sum of an integer and its opposite results in zero, emphasizing the concept of balance in mathematical expressions. Through exercises and practice, students can develop a deeper comprehension of integer operations and their symbolic representations.

Table 1. Diagram illustrating the educational activity on integers.

Level	Level Characteristics, Meaning, Content	Activity Implementation Scope	General Overview of Level Activities
I	Demonstration through concrete actions	Physical whole perception	Transform the mathematical meaning and content of positive, negative, opposite numbers, zero, and their operational characteristics into concrete meaningful actions, making them concrete or observable in reality. At this level, creatively use symbolic counters. Establish that counting is the fundamental didactic activity for whole numbers.

II	Visual representation	Mental imagination and visualization	At the representational level, transform the whole physical sensations of the mathematical meaning and content of positive, negative, opposite numbers, zero into imagination and represent them through images. At this level, creatively use the number line. While introducing whole numbers and their operational characteristics through the number line, establish that counting is the fundamental didactic activity for whole numbers.
III	Symbolic notation and summarization	Meaning assignment, summarization, and coding	At the symbolic level, convert the whole sensations, mental representations, drawings, and visualizations of the mathematical meaning and content of positive, negative, opposite numbers, and zero into mathematical language symbols and summarize them symbolically. At this level, make students understand that zero is empty, but it represents the sum of two opposite numbers. While everything may seem empty when there is no empty condition, whenever we perceive it mathematically and write mathematical code, that emptiness is actually not empty, but rather the sum of two opposite numbers.

This appears to be an educational framework describing three levels of mathematical learning progression:

1. **Level I** focuses on concrete, hands-on learning where students physically manipulate objects to understand mathematical concepts.
2. **Level II** involves visual and mental representation, using tools like number lines to help students visualize mathematical ideas.
3. **Level III** deals with abstract symbolic notation, where students learn to represent mathematical concepts using symbols and mathematical language.

The document specifically addresses teaching positive numbers, negative numbers, opposite numbers, and zero, emphasizing how students' progress from concrete understanding to abstract mathematical thinking. It's particularly detailed about teaching the concept of zero as the sum of two opposite numbers rather than simply "emptiness."

This educational approach, which utilizes the concepts of positive, negative, opposite numbers, and zero, provides a simple and effective method for children to grasp fractions and arithmetic operations in an engaging manner. By integrating the principles of natural number pedagogy with negative numbers, opposites, and zero, students can easily comprehend complex mathematical expressions. For instance, dividing 1 by the opposite of 1/2 results in 2, as it represents the opposite of two. Similarly, multiplying the opposite of three by the opposite of four yields -0.3. Understanding the concept of adding one hundredth twelve times with a negative sign in the expression -0.4 simplifies the calculation to one hundredth. By mastering the fundamentals of integers through positive, negative, opposite numbers, and zero, students can enhance their understanding of all numerical operations.

When using a symbol counter to teach integers, students quickly grasped the lesson, avoided symbol errors, and showed interest in problem-solving. Parents were encouraged to observe the lesson and provide feedback. After the lesson, students were asked about their experience with the symbol counter method, and parents noted that it was a unique, engaging, and effective approach that helped students comprehend the material without making symbol errors. In contrast, when taught using traditional methods, students struggled with symbol errors and took longer to grasp the concepts.

The students found the lesson easy to grasp and engaging. Time flew by, and the students found it enjoyable, interesting, and comprehensible. The teacher's effective instruction turned knowledge into practical skills. Parents, school administrators, and teachers were present at the lesson. Representatives of the teachers noted that one positive aspect was the absence of errors in grading. They also observed that the students were attentive and each child was able to work independently during the lesson. This study contributes to the advancement of mathematics education by emphasizing the value of innovative, student-centered methodologies that make abstract concepts accessible and meaningful.

Conclusions

This experimental study has confirmed the effectiveness of methodological approaches that enhance student engagement, critical thinking, self-directed learning, and collaboration in mathematics education. The focus was on utilizing the symbol counter methodology to teach integer operations, addressing common challenges students encounter with sign calculations and conceptual understanding.

The results of the experiment show that the symbol counter approach significantly improves student learning outcomes in key areas:

- Students using the symbol counter method showed notable improvement in understanding positive and negative numbers, opposite numbers, and zero. The three-level pedagogical framework, progressing from concrete manipulation to visual representation and symbolic notation, proved highly effective in building conceptual understanding.
- A significant achievement was the substantial decrease in sign calculation errors. Traditional teaching methods often led to frequent mistakes in operations like (-) × (-) = (+), which the symbol counter method eliminated by providing concrete, visual representations of abstract concepts.
- Student engagement levels were significantly higher during lessons employing the symbol counter methodology. The interactive nature of the approach transformed abstract mathematical concepts into tangible experiences, maintaining student interest throughout the learning process.
- Students demonstrated enhanced logical thinking and problem-solving abilities when working with integer operations. Emphasizing zero pairs and visualizing mathematical relationships fostered deeper analytical thinking.

The research confirms that the three level didactic approach practical application, visual representation, and symbolic notation offers a comprehensive framework for teaching integers effectively. This methodology bridges the gap between concrete understanding and abstract mathematical thinking, making complex concepts accessible to all learners.

The study also emphasizes the importance of using manipulative tools in mathematics education. The symbol counter not only serves as a teaching aid but also acts as a cognitive bridge that helps students internalize mathematical relationships and develop number sense.

This research contributes to the growing evidence supporting hands-on, manipulative based approaches in mathematics education. The success of the symbol counter method demonstrates that complex mathematical concepts can be made accessible through well-designed pedagogical tools and structured learning progressions.

The study's success with integer operations suggests potential applications for other challenging mathematical topics, such as fractions, algebraic expressions, and advanced arithmetic operations. The foundational understanding developed through this method appears to enhance students overall mathematical competency.

Recommendations for Implementation

Based on the experimental outcomes, we recommend:

1. Educators should undergo comprehensive training on the three-level pedagogical approach and symbol counter methodology for effective implementation.

2. The symbol counter method should be systematically integrated into mathematics curriculum, especially for integer operations and related topics.

3. Schools should invest in appropriate manipulative materials and visual aids to support this methodology.

4. Regular evaluation of student progress and method effectiveness should be conducted to ensure sustained benefits.

The experimental validation of the symbol counter methodology represents a significant advancement in mathematics pedagogy. By addressing fundamental challenges in teaching integer operations, including sign errors, conceptual misunderstandings, and student disengagement, this approach offers a practical, effective solution for educators globally.

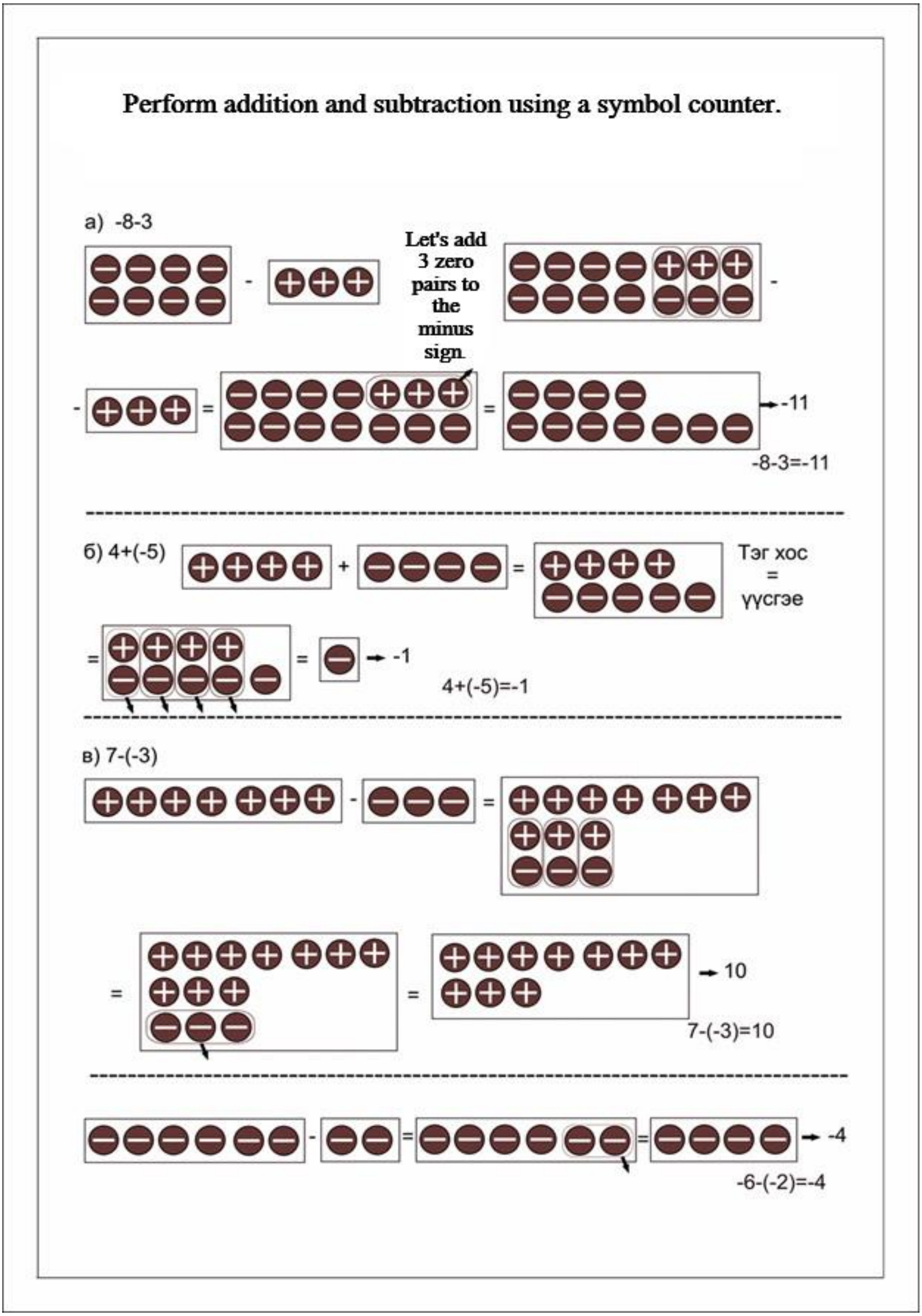
The research demonstrates that providing students with concrete, visual, and meaningful ways to explore mathematical concepts deepens their understanding, reduces errors, and increases engagement. This study not only validates the specific methodology tested but also reinforces the broader principle that effective mathematics education requires innovative, student-centered approaches that make abstract concepts tangible and comprehensible.

The successful implementation of this methodology in experimental settings, along with positive stakeholder feedback, lays a strong foundation for broader adoption and continued research in mathematics education innovation.

REFERENCES

1. Batsuuri, Ts. *Education for sustainable development: Lectures*.
2. Doyod, U., & Dalaijamts, Ts. (2008). *Methodology for supporting learning through learning activity evidence*. Ulaanbaatar.
3. McKay, A., & Erdene, L. (2007). *Mathematics - Grade 9 textbook*. Ulaanbaatar.
4. *Mathematics education standard* (2005). Ulaanbaatar.
5. *Mathematics education standard guidelines* (2003). Ulaanbaatar.
6. Batsuuri, Ts. (2006). *Issues in educational technology innovation* (p. 13). Ulaanbaatar.
7. Ichinkhorloo, Sh. (2013). *Learning theory and application: Foundations of general didactics*. Ulaanbaatar.
8. Nyamjav, Ts., & Bunden, D. (2005). *Mathematical didactics*. Ulaanbaatar.
9. Adyaasuren, V., & Ariunzaya, G. (2011). *Mathematics - Grade 9 textbook*. Ulaanbaatar.
10. *Mathematics Grade 6* (2013). Ulaanbaatar.
11. *Mathematics Grade 9* (2016). Ulaanbaatar.
12. *Mathematics Grade 10* (2013). Ulaanbaatar.
13. *Mathematics Grade 11* (2013). Ulaanbaatar.
14. *Mathematics Grade 12* (2015). Ulaanbaatar.
15. *Mathematics Grade 5* (2016). Ulaanbaatar.
16. *Mathematics and natural science didactics: Key issues, solutions, and approaches* (2015). Ulaanbaatar.
17. Sandagdorj, B. *Research on the origin and development of mathematics Olympiad and its problem system* [Doctoral dissertation]. Ulaanbaatar.
18. Javzandulam, Sh., & Jugdernamjil, B. (2016). *Guide to writing academic papers according to the "ARA" model*. Ulaanbaatar.
19. Aman, S., & Oranshtai. (2001). *Active learning strategies*. Ulaanbaatar.
20. Kolmogorova, A. N., & Abramov, A. M. *Algebra and beginning analysis Grades 9-10*.
21. Bert Creemers, Leonidas Kyriakides, & Panayiotis Antoniou. *Teacher development in improving teaching quality*. London.
22. Leontieva, M. R., & Muravin, K. S. *Didactic materials in algebra for Grade 6*.
23. Robert A. Reiser, & John V. Dempsey. *Instructional design and technology: Trends, issues, and approaches*.
24. Menchinskaya, N. A. (1972). *Psychological foundations of learning* (p. 73).
25. Stolyar, A. A. (1981). *Methods of mathematics teaching*.
26. Ministry of Education, Culture, Science and Sports. (2013). *Report on developing requirements and cost calculation methodology for digital learning objects used in general education school subjects* (p. [page number]). Asian Development Bank.
27. Luvsandorj, Ts. & Munkhtaria, Kh. (2016). *Integer fundamentals didactics*. Ulaanbaatar.
28. Luvsandorj, Ts. & Munkhtaria, Kh. (2017). *The art of teaching: Mathematics lesson demonstrations*. Ulaanbaatar.
29. Luvsandorj, Ts. & Munkhtaria, Kh. (2017). *Mathematics lesson demonstrations: Volume one*. Ulaanbaatar.
30. Munkhtaria, Kh. & Luvsandorj, Ts. (2015). *Natural number fundamentals didactics*. Ulaanbaatar.
31. Illich, I. (1995). *In the graveyard of the text: A commentary to Hugh's Didascalicon*. University of Chicago Press.
32. Luvsandorj, Ts. (2009). Towards reconsidering strategies for ensuring gender equality in education in the light of neuroscience: Either equality through difference or equality through sameness or neither 'through difference' nor 'through sameness'? Critical review. Retrieved from http://mr-institute.blogspot.com
33. Mäntylä, T. (2011). *Didactical reconstructions for organizing knowledge in physics teacher education*. Retrieved from http://ethesis.helsinki.fi/
34. Comenius, J. A., Locke, D., Rousseau, J.-J., & Pestalozzi, I. G. (1989). *Pedagogical heritage* (416 pp.). Pedagogy. (ISBN 5-7155-0164-4)
35. Wiley, D. A. Connecting learning objects to instructional design theory: A definition, a metaphor, and a taxonomy (pp. 22-24). Utah State University.

APPENDIX


Perform addition and subtraction using a symbol counter.
а) -8-3
Let's add 3 zero pairs to the minus sign.
-11
-8-3=-11
б) 4+(-5)
Тэг хос = үүсгэе
-1
4+(-5)=-1
в) 7-(-3)
10
7-(-3)=10
-4
-6-(-2)=-4

COPYRIGHT CERTIFIICATE

MONGOLIA

COPYRIGHT

CERTIFICATE

Number 8169

Name of the work

"Signal counter"

Date of creation

October 15, 2015

Type of work

Dictionaries, reference books, anthologies, and other compilations of data and databases

The owner of the work

Khayankhyarvaa MUNHTARIYA, Tsogdov LUVSANDORJ

Creators

Khayankhyarvaa MUNHTARIYA, Tsogdov LUVSANDORJ

Registered in the state register by order No. A/287 of the Director of the General Department of Intellectual Property and State Registration of Mongolia dated November 2, 2016.

CHIEF R. SODHUKHU

Ulaanbaatar city