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# KEY ASPECTS OF USING NUMERICAL METHODS IN THE MECHANISM OF INVESTMENT OF SMALL AND MEDIUM-SIZED BUSINESS ENTERPRISES

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**ABSTRACT**

In today's realities of investment activities, small and medium enterprises are faced with the dilemma of choosing areas of investment of monetary and non-monetary resources. Determining the feasibility of a particular investment is based on the use of appropriate mathematical apparatus and economic-mathematical models. The article considers some of the most important aspects of the use of numerical methods in the mechanism of investment of small and medium enterprises, developed on the basis of research on the development of such economic entities. The logical basis of the proposed methods is a proven division of the term of operation of enterprises into several stages of the life cycle, which differ significantly in the characteristics of incoming and outgoing cash flows.

It is proposed to determine the absolute expediency of founding small and medium enterprises by calculating the probabilistic values of profitability of the enterprise based on the calculation of the area of geometric shapes limited by the functions of income and expenses during all stages of the life cycle. It is proved that such calculations are expedient by calculating the indefinite integral and applying the Newton-Leibniz formula.

With regard to the functions that characterize the income and expenses of small and medium enterprises throughout their existence, the article proves that the most optimal in this case is the use of production functions. Among the whole set of existing production functions, we have identified the most optimal for the task Cobb-Douglas production function, CES production function, Linear production function, Leontief production function, adapted to the conditions of the microlevel.

To calculate the elementary areas limited by the corresponding production functions, the article examines the relevant existing methods. Studies have been conducted on the possibility of using to calculate the areas limited by the production functions of income and expenses of small and medium enterprises during each stage of the life cycle, the methods of rectangles, trapezoids and parabolas. It is determined that they have limited conditions for use due to the need to divide the time period of operation of enterprises at intervals that do not coincide with the stages of the life cycle. Therefore, studies were continued, as a result of which it was proved that the most optimal method of calculating the elementary areas limited by the target functions of income and expenditure of small and medium enterprises is the Gaussian method with different variations.

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**Introduction and problem statement.** To determine the feasibility of establishing and operating small and medium-sized businesses, it is necessary to examine its financial aspects of the investment mechanism of the relevant business entity. First of all, any investor is interested in the return on his investment, the next question is the possible size of the projected profit. The establishment of such a business in principle depends on the answer to these questions. Therefore, the definition of such parameters of small and medium enterprises is very relevant and necessary.

**Analysis of recent research and publications.**

The analysis of literary scientific sources has shown that to date the issue of using numerical methods in the investment mechanism of small and medium enterprises is considered rather weakly and superficially. Of course, there are quite thorough works in this area, which we used in our own research [1, 4, 5], but they all consider only some aspects of the problem, without giving a comprehensive solution.

**Forming the purpose of the article.** The purpose of this article is to develop methodological approaches to identify key aspects of the use of numerical methods in the mechanism of investment of small and medium enterprises.

**Research results.** We have already proved [3] that small and medium-sized businesses go through a number of stages of the life cycle unique to them. Each of the stages is characterized by different amounts of financial income and expenses [2].

Figure 1 shows a schematic representation of financial security in terms of financial flows of income and expenses of each stage of the life cycle of small and medium enterprises. So, if we consider this figure only from the mathematical point of view, we see two curves that can be considered as certain functions that are the boundaries of certain planes. The area of the figures bounded by the abscissa, the constraints on the abscissa and the curves of the functions of income and expenses, is essentially the size, respectively, income and expenses for the entire period of existence of such business entities.

However, it is clear that in this form in reality it is impossible to generate income and expenses due to the fact that they are carried out unevenly. That is, costs and revenues are discrete in nature. It would be more correct and accurate to display the data shown in Fig. 1, in the form of diagrams, as a result of which we would obtain a piecewise linear function. However, such an approach would not allow for further calculations. Therefore, we consider it expedient in this case to use a function smoothed by curvilinear trapezoids, which to a greater extent demonstrates the trend of the event. It is the introduction of such an approach that allows for numerical calculations using a quantitative assessment of the trend of profit and expenditure. The application of smoothing is necessary to determine the trend and the possibility of applying modeling techniques.

Therefore, in order to determine the feasibility of the establishment and continued existence of the enterprise, it is necessary to calculate the area of these figures and from the value of income to deduct the value of costs. In the case of a positive value, it is possible to draw conclusions about the absolute feasibility of establishing and further functioning of small and medium enterprises. Absolute expediency will show us the hypothetical existence of profits. However, of course, more informative is the comparative feasibility, which shows the profitability of activities compared to other similar projects. To obtain such data, it is necessary to make appropriate calculations for other projects.

Therefore, in any case, it is necessary to solve the mathematical problem of calculating the probabilistic values of profitability of the enterprise on the basis of calculating the area of the figures described above. According to the theoretical approaches of higher mathematics, this solution is possible by applying the definite integral, which in turn is determined by calculating the indefinite integral and applying the Newton-Leibniz formula. Thus, we have the following mathematical expression:

$$VP = VI - VC = \int_{a}^{в} f_{income}(t)dt - \int_{c}^{в} f_{costs}(t)dt \quad , \qquad (1)$$

where $VP$ – the amount of profit for the entire period of activity of small and medium enterprises;
$VI$ – the amount of income for the entire period of activity of small and medium enterprises;
$VC$ – the amount of costs for the entire period of activity of small and medium enterprises;
$a$ – the beginning of the start stage;
$в$ – the end of the liquidation stage;
$c$ – the beginning of the stage of birth of the idea;
$f_{income}(t)$ - income function;
$f_{costs}(t)$ - cost function.

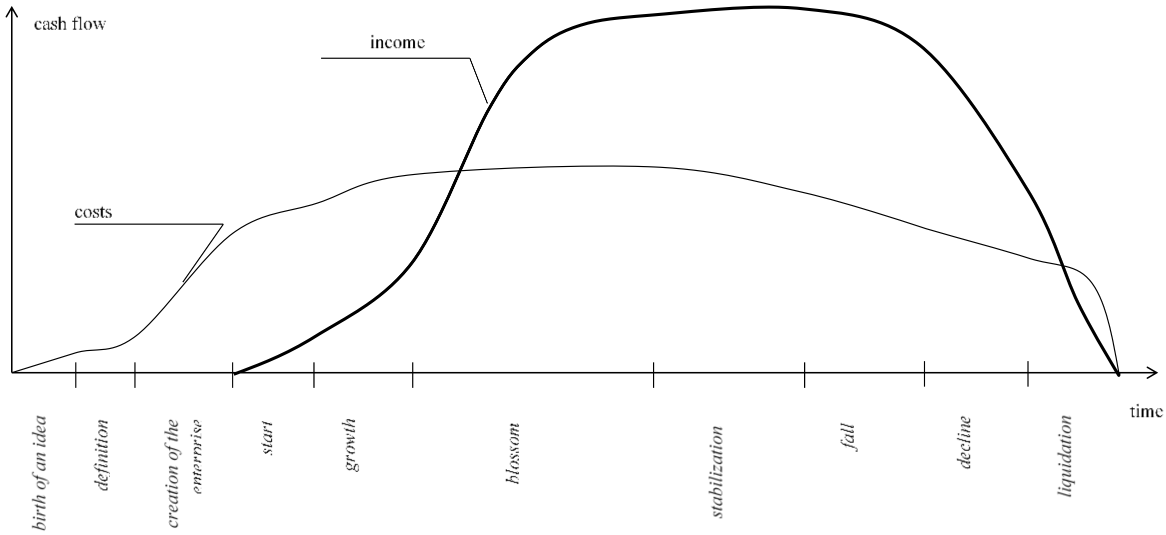


*Fig. 1. Schematic representation of financial provision in terms of financial flows of income and costs of each stage of the life cycle of small and medium enterprises*

Continuing the mathematical solution of expression (1) we have the following:

$$VP = \lim_{\Delta t \to 0} \sum_{i=0}^{n-1} f_{income}(\xi_i)\Delta t_i - \lim_{\Delta t \to 0} \sum_{j=0}^{n-1} f_{costs}(\delta_j)\Delta t_j \quad (2)$$

The next step in our study is to determine the type of income and expense functions. In our opinion, the production function, which we will describe in detail, is the most suitable for this. The issue of the function that characterizes the amount of income and expenses during the entire period of operation of small and medium enterprises should be studied separately.

The analysis of literature sources showed that the most optimal variant of such a function is the production function, taking into account the stages of the life cycle of the enterprise. In the classical form, the production function is interpreted as the dependence of the result of production activities on its determining factors. A separate case of the production function is the function of output, which is interpreted as the dependence of production on the availability or consumption of resources, and the cost function - the dependence of production costs on the volume of manufactured products [4].

According to the works of Samochkin V. M. [5], in which he carefully examines the activities of enterprises precisely at the stages of their life cycle, it can be noted that the production functions, depending on the state of the enterprise and a number of circumstances may be:

1. Cobb-Douglas production function, which provides for constant elasticity of output by factors of production:

$$Y = A \cdot L^{\lambda} \cdot K^{1-\lambda}$$

2. Production function CES (Constant elasticity of substitution), having a constant substitution elasticity: $Y = A \cdot \left((1-\alpha) \cdot K^{-\rho} + \alpha \cdot L^{-\rho}\right)^{-\frac{\beta}{\rho}}$

3. Linear production function: $Y = aK + bL$.

4. Leontiev's production function adapted to microlevel conditions: $Y = min\left(\frac{K}{a}, \frac{L}{b}\right)$.

That is, among all the sets of such production functions, you should choose the one that is most suitable in a particular case and takes into account all aspects of the enterprise.

As practical calculations show, the choice of the type of production function is a very important and responsible task, because the final result of all analytical work significantly depends on it. Therefore, we propose to choose to use a number of methodological developments in this regard, one of which is a textbook Dudov S.I. and others [1], which distinguishes several types of production functions.

1. Typical production function. It is used in the case of production of only one type of product. Of course, this situation is not common and is more typical of various enterprises that are natural monopolists, but still this type of production functions is the starting point for the implementation of appropriate calculations.

Therefore, for this situation, n types of costs are considered and a cost vector is formed, which has the following form:

$$x = (x_1, x_2, \ldots, x_n)^T \tag{3}$$

where $x_i$ – quantity of the *i*-th type of costs of the enterprise.

The set of all possible cost vectors, assuming that all costs can change continuously, is called the cost space. Due to the fact that all costs are non-negative quantities, the cost space can be considered as an integral ortant of Euclidean space, which can be calculated by the following formula:

$$T = \left\{ x \in R^n : x_i \geq 0, i = \overline{1,n} \right\}, \tag{4}$$

The production function in this case is called the functional relationship between costs and output.

The production function can characterize the constant (proportional) growth of income from increasing production and declining or increasing (disproportionate) growth of income from increasing production. In the first case, the production function can be written as follows:

$$f(\alpha x) = \alpha f(x) \quad at \quad \alpha > 1, \tag{5}$$

$$\alpha x = (\alpha x_1, \alpha x_2, \ldots, \alpha x_n)^T.$$

In the second case, the production function is as follows:

$$\begin{gathered} f(\alpha x) > \alpha f(x) \\ \left( f(\alpha x) < \alpha f(x) \right). \end{gathered} \tag{6}$$

In order to determine the sensitivity of the production function to changes in the parameters of the use of costs, mathematicians propose to calculate the elasticity of output relative to changes in a certain type of costs according to the following formula [1]:

$$\varepsilon_i(x) = \frac{x_i}{f(x)} \cdot \frac{\partial f(x)}{\partial x_i}, i = \overline{1,n}. \tag{7}$$

In the practice of many companies very often there is a question of replacing some materials, raw materials, etc. with others like it. In order for such substitution not to be reflected in the rhythm of production, it is necessary to calculate the marginal rate of substitution of one (*i*-th) resource for another (*k*-th):

$$s_{ik}(x) = \left. \frac{\partial f(x)}{\partial x_i} \middle/ \frac{\partial f(x)}{\partial x_k} \right., \quad i,k = \overline{1,n}. \tag{8}$$

This parameter shows how much you need to spend additional *k*-th resource instead of the *i*-th, so that the estimated value of the production function does not change.

These mathematical calculations are very simple and clear, but in practice they are practically not used. This is due to the fact that, as noted above, this typical production function is used only in the case of production of only one type of product. In reality, this is almost non-existent. Even monopoly enterprises in the case of production of one type of product provide a number of related services, which makes it impossible to use this type of production functions in general, and especially for small and medium enterprises.

More practical and realistic for application is the neoclassical formation of the production function based on the formation of the cost vector and the price vector of sales of all types of products, works and services using the Kuhn-Tucker theorem, Sylvester criterion and Slater's conditions. Very often in such cases the Lagrange function and its derivatives should be used.

The next question is the mathematical methods of calculating the area of curved trapezoids, which are limited by the target functions of income and expenditure of small and medium enterprises.

As we wrote above, the calculation of the area of such trapezoids, equal to the amounts of income and expenses of the enterprise throughout their existence, is possible by applying integrated methods of calculation.

We have already proved that it is most expedient to consider the corresponding production function as a subintegral function. If we replace the subintegral function with a zero polynomial, or a polynomial of the first and second degree, then the method of rectangles, trapezoids and parabolas will be the most optimal for calculating the area of a curvilinear trapezoid. In the classical form, these methods are problematic to apply, so we will try to adapt them to our conditions and the needs of determining the feasibility of small and medium enterprises.

We will be the first to consider and adapt the method of rectangles to our needs. Graphic interpretation of its essence is shown in Fig. 2.

The whole plane of a curvilinear trapezoid is divided into a number of rectangles and their area is calculated. That is, we have the following expression:

$$\int_a^b f(x)dx \approx \frac{b-a}{n}(y_0 + y_1 + \ldots + y_{n-1}). \tag{9}$$

where $a, b$ – the boundary limits of the segment of the subintegral function, the area of which must be determined;

$x_0, x_1, \ldots, x_n$ – points by which the space from point $a$ to point $b$ is divided at equal intervals.

$y_0, y_1, \ldots, y_{n-1}$ – the values of the subintegral function (in our case the production function) respectively at the points $x_0, x_1, \ldots, x_{n-1}$.

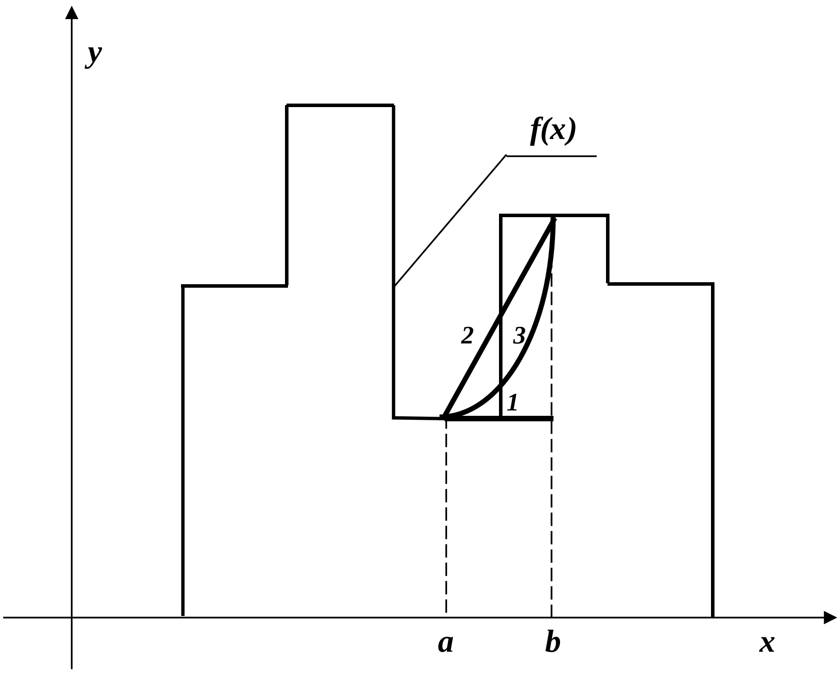


*Fig 2. Graphical interpretation of numerical integration methods.*

In the figure: $f(x)$ – a discrete function; $[a, b]$ – segment on the abscissa axis; 1, 2, 3 – graphical interpretation of numerical integration by the method of rectangles, trapezoids, parabolas (Simpson's method), respectively.

Thus, we see that in the classical form, this method can not be used for our task precisely because of the need to divide the time space of small and medium enterprises at intervals. As shown by our own research [2, 3], which are partially shown in Fig. 1, the stages of the life cycle of small and medium enterprises are different in duration, and therefore require the use of another mathematical apparatus.

The next method, which is quite popular in such calculations, is the trapezoidal method. Its essence is to calculate the area of each trapezoid, which is limited on the sides by time intervals, and above - the production function. The area of the trapezoid on each segment, and in our case such a segment is the stage of the life cycle, is calculated by the following formula:

$$I_i \approx \frac{f(x_{i-1}) + f(x_i)}{2}(x_i - x_{i-1}). \tag{10}$$

Or when dividing the entire integration interval at the level of segments of the same length h, you can use the following formula:

$$I \approx h\left(\frac{f(x_0) + f(x_n)}{2} + \sum_{i=1}^{n-1} f(x_i)\right), \text{ where } \; h = \frac{b-a}{n} \tag{11}$$

This method, in contrast to the previous method of rectangles, is more accurate. However, it also does not quite fit our needs precisely because of the need to divide the intervals at the level of integration intervals.

There is also the parabola method (Simpson's method), but it also works only if there are equal segments of integration.

Therefore, it can be noted that the above methods of integration are appropriate for use only in some cases. If the stages of the life cycle of a particular enterprise are equal, they can be used. However, this situation with equal intervals is most likely an exception to the rule, and therefore it is necessary to continue the search for other methods of calculating the area of the figure, which limits the production function.

Among the whole set of more complex methods, in our opinion, the most optimal for solving the problem is the Gaussian method and its variations.

The Gaussian method basically has points at which it is possible to calculate the production function, thereby dividing the integration interval into segments of different lengths. In addition, due to the use of a more complex mathematical apparatus, this method allows calculations with a higher third order of accuracy. For the two values of the production function that limit the stages of the life cycle of the enterprise, we have the following formula for calculating the area of the trapezoid:

$$I \approx \frac{b-a}{2}\left( f\left(\frac{a+b}{2} - \frac{b-a}{2\sqrt{3}}\right) + f\left(\frac{a+b}{2} + \frac{b-a}{2\sqrt{3}}\right)\right). \tag{12}$$

The Gaussian method has a number of variations that should be applied in different situations. These are the Gauss-Kronrod method, the Chebyshev method, the Monte Carlo method and the Runge-Kutti method. Each of them has its own characteristics, but all of them under certain conditions, determined by the specific circumstances of certain small and medium enterprises, can be used to solve the problem set in our study.

**Conclusions.** So, summarizing all the above, we can say that determining the return on investment and calculating the possible size of the projected profit requires the use of a serious mathematical apparatus. In our opinion, the answer about the absolute feasibility of investment can be obtained by calculating the difference between expected income and expenses. Projected revenues and expenses can be obtained by calculating the area of curved trapezoids bounded by the corresponding production functions. Our literature studies have shown that the most optimal production functions in our case are the Cobb-Douglas production function, the CES (Constant elasticity of substitution) production function, the linear production function, and the Leontief production function adapted to microlevel conditions. The direct choice of the type of production function is made depending on the specific conditions of the practical situation. The next issue that was investigated is the choice of methods for calculating the area of curved trapezoids, which are limited by the target functions of income and expenditure of small and medium enterprises. Among all the existing methods, we have proved that the most optimal for solving this problem is the Gaussian method with different variations.

Thus, the application of the proposed methodological aspects of determining the feasibility of investment in small and medium-sized businesses is a necessary condition for effective management of business entities.

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